Homework 9, Morally due Tue Apr 23, 3:30PM

THIS HW IS TWO PAGES!!!!!!!!!!

1. (40 points) Throughout this problem Bill has a 2-sided dice with numbers 1,2 and a 3-sided die with numbers 1,2,3.

   (a) (15 points) Assume both dice are fair. Bill throws both of them. For $2 \le i \le 5$ give the prob that the sum is i.

   (b) (20 points) Let $0 \le p \le \frac{1}{2}$. Assume the 2-sided dice is fair but the 3-sided dice has

   Prob of $1 = p$

   Prob of $2 = 1 - 2p$

   Prob of $3 = p$

   Bill throws both of them. For $2 \le i \le 5$ give the prob that the sum is i.

   (c) (5 points) Let p be as in the last part. Is there a value of p such that all of the sums $2, 3, 4, 5$ come up with the same probability.

   (d) (0 points but thing about it) Can you load two 6-sided dice to get fair sums?

SOLUTION TO PROBLEM ONE

1) Both dice are fair.

Prob(2) = Prob of (1,1) $= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Prob(3) = Prob or either (1,2) or (2,1) $= 2 \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}$.

Prob(4) = Prob or either (1,3) or (2,2) $= 2 \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}$.

Prob(5) = Prob or (2,3) $= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

2)

Prob(2) = Prob of (1,1) $= \frac{1}{2} \times p = \frac{p}{2}$.

Prob(3) = Prob or either (1,2) or (2,1) $= \frac{1}{2} \times (1 - 2p) + \frac{1}{2}p = \frac{1-p}{2}$.

Prob(4) = Prob or either (1,3) or (2,2) $= \frac{1}{2} \times p + \frac{1}{2} \times (1 - 2p) = \frac{1-p}{2}$.

Prob(5) = Prob or (2,3) $= \frac{1}{2} \times p = \frac{p}{2}$.

3) If $p = 1/2$ then all of the probabilities are $\frac{1}{4}$.

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2. (60 points) On the planet Vorlon they play a game that is similar to what we call Poker but with a different deck of cards.

   Every card has a rank from $\{1, 2, \ldots, 7\}$.

   Every card has a suite from $\{R, B\}$.

   Every player gets 3 cards.

   In most of the questions we will ask for the prob of a certain type of hand. Give the answer to 4 places since the last question is to rank them.

   (a) What is prob of a straight that is NOT a flush (e.g., $3R$, $4R$, $5B$) We DO allow wrap-around, so 7-1-2 counts.

   (b) What is prob of a flush that is NOT a straight (e.g., $2R$, $4R$, $9R$)

   (c) What is prob of a straight flush (e.g., $3R$, $4R$, $6R$) We DO allow wrap-around, so 7-1-2 counts.

   (d) What is prob of a pair (e.g., $3R$, $3B$, $7R$). Note that a pair cannot be a straight of a flush.

   (e) What is prob of getting NOTHING- a hand that is neither a straight, nor a flush, nor does it contain 2 of a kind. (e.g., $3R$, $5R$, $6B$)

   (f) Rank the types of hands from most likely to least likely.

   SOLUTION TO PROBLEM TWO

   Note that the total number of hands is $\binom{14}{3} = 364$.

   (a) A straight that is NOT a flush.

   Pick a rank r — there are 7 ways to do this. Then you have $r, r+1, r+2$. Now pick for each card R or B, but DO NOT pick RRR or BBB so you pick one of 6 R-B sequences. So 42. So prob is $\frac{42}{364} = \frac{3}{26} \sim 0.12$. NOTE FOR LATER: 42 ways to get a straight, NOT a flush.

   (b) A flush that is NOT a straight (e.g., $2R$, $4R$, $9R$)

   Pick a suit — there are 2 ways to do this. Then pick 3 ranks — there are $\binom{7}{3}$ ways to do that. NO- need to make sure they are not a straight. There are 7 straights: 123, 234, ..., 712. So prob is $2 \times (\binom{7}{3} - 7) = 2(35 - 7) = 2 \times 28 = 56$. So Prob is $\frac{56}{364} \sim 0.154$. NOTE FOR LATER: 56 ways to get a flush, NOT a straight.

(c) A straight flush.

Pick a rank — there are 7 ways to do this. Pick a suite — there are 2 ways to do this. So there are $7 \times 2 = 14$ ways to get a straight flush. So prob is $\frac{14}{364} \sim 0.038$.

NOTE FOR LATER: 14 ways to get a straight flush.

(d) A pair.

Pick a rank — there are 7 ways to do this. The suits are determined-one will be R and one will be B. Then pick the other card — there are $14 - 2 = 12$ ways to do this. So there are $7 \times 12 = 84$ ways to get a pair. So prob is $\frac{84}{364} \sim 0.23$.

NOTE FOR LATER: 84 ways to get a pair.

(e) NOTHING.

All of the above types are disjoint. Hence we need only subtract. The number of hands with NOTHING is

$364 - 42 - 56 - 14 - 84 = 168$. So the prob of getting nothing is $\frac{168}{364} \sim 0.46$.

(f) RANK: from most likely to least likely:

NOTHING: Prob ~ 0.46.

A PAIR: Prob ~ 0.23.

FLUSH THAT IS NOT A STRAIGHT: Prob ~ 0.154.

STRAIGHT THAT IS NOT A FLUSH: Prob ~ 0.12.

STRAIGHT FLUSH: Prob ~ 0.038.