

Homework 12, Morally due Tue May 14, 3:30PM

**THIS HW IS ONE PAGES!!!!!!!!!!**

WHEN IS THE FINAL? Saturday May 18, 4-6

WHERE IS THE FINAL? PHYSICS 1201

1. (30 points — 10 each) Show that the following sets are uncountable
  - (a) The set of functions from  $\mathbf{N}$  to  $\mathbf{N}$  that are strictly increasing. (That means that, for all  $x, y \in \mathbf{N}$ , if  $x < y$  then  $f(x) < f(y)$ .)
  - (b) The set of functions from  $\mathbf{N}$  to PRIMES.
  - (c) The set of functions from  $\mathbf{N}$  to PRIMES that are strictly increasing.
  
2. (40 points — 20 points each) Let  $(A, \leq_1)$  and  $(B, \leq_2)$  be ordered sets. An *order preserving bijection*  $f$  from  $A$  to  $B$  is a bijection from  $A$  to  $B$  such that, for all  $x, y \in A$ .

$$x \leq_1 y \rightarrow f(x) \leq_2 f(y).$$

- (a) Show that there is NO order preserving bijections from  $\mathbf{N}$  to  $\mathbf{Z}$ .
  - (b) Show that there is NO order preserving bijections from  $\mathbf{N}$  to  $\mathbf{Q}^{\geq 0}$ . (Thats the rationals  $\geq 0$ .)
  
3. (30 points) Prove or disprove: If  $A_1, A_2, \dots$  are countable and disjoint then  $A_1 \times A_2 \times A_3 \times \dots$  is countable.