CMSC 250 Final

1. This is an open-everything exam. You can use anything except ask another person. **Caution:** if you copy from the web or elsewhere mindlessly you will probably get it wrong.

2. There are 3 problems which add up to 70 points. Recall that you already did 30 points of this midterm take home.

3. The exam is Monday May 17, 8:00PM-10:15PM unless you have contacted me to make other arrangements. So the exam is 2 hours and 15 minutes

4. For each question show all of your work and **use LaTeX or write VERY NEATLY**. **Clearly indicate** your answers. No credit for illegible answers.

5. Please write out the following statement: *I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*

1. (20 points) Prove that $11^{1/4}$ is NOT rational. You must state and prove carefully any lemmas you use. (Do the problem in the style we did such proofs in class. That is, do not do a prove that uses Unique Factorization, a technique we did not do in class, so do not worry if you don't know what that means.)

   You can do this problem on this page and the next page.

2. (20 points- 5 points each)

   We will use the following definitions.

   Definition Let f be a function.

   f is *strictly increasing* if, for all $x < y$, $f(x) < f(y)$.

   f is *monotone increasing* if, for all $x < y$, $f(x) \le f(y)$.

   f is *strictly decreasing* if, for all $x < y$, $f(x) > f(y)$.

   f is *monotone decreasing* if, for all $x < y$, $f(x) \ge f(y)$.

   And now FINALLY the problem

   For each of the following sets say whether it is (1) finite, (2) countable, or (3) uncountable and PROVE it.

   (A set X is **countable** if there is a BIJECTION to N.)

   (a) The set of all functions f with domain N and codomain N that are strictly increasing.

   (b) The set of all functions f with domain N and codomain N that are strictly decreasing.

   (c) The set of all functions f with domain N and codomain Z that are monotone increasing.

   (d) The set of all functions f with domain N and codomain Z that are monotone decreasing.

   You can do this problem on this page and the next page.

3. (30 points-10 points each) Let

$$A = \{1, \ldots, 100\}$$

and

$$B = \{1, \ldots, 200\}.$$

For each of the following sets say how big they are.
(You may use the following notations:
factorial, e.g., 10!
choose, e.g., $\boldsymbol{\binom{19}{9}}$
things like $\mathbf{5 \times 6 \times \cdots \times 120.}$
)

(a) The set of all functions with domain A and co-domain B.
(b) The set of all 1-1 functions with domain A and co-domain B.
(c) The set of all 1-1 functions with domain B and co-domain A.