Take-Home Final Problems,
Morally due Mon May 10, 9:00AM, Dead Cat Wed May 12 9:00PM (YES PM!)

On Monday May 17, 8:00PM-10:15PM is your final. It will be 2.25-hours and be 70 points. Why 70 points? Because THIS document has THREE problems that you will do ahead of time and have plenty of time to do, which is worth 30 points.

This must be handed in typed and easy to read.

1. (10 points) All quantifiers are over N. Let $Q(n)$ be the statement

$$(\exists d_1 \le \cdots \le d_n)\left[1 = \sum_{i=1}^{n} \frac{1}{d_i^2}\right]$$

(Note that the d_i's do NOT need to be distinct.)

   (a) (1 points) Prove that $Q(5)$ is FALSE.

   (b) (1 points) Prove $Q(6)$ is true by giving 6 numbers $d_1, \ldots, d_6$ such that $1 = \sum_{i=1}^{6} \frac{1}{d_i^2}$.

   Hint Write a program to find the numbers for your.

   (c) (1 points) Prove $Q(7)$ is true by giving 7 numbers $d_1, \ldots, d_7$ such that $1 = \sum_{i=1}^{7} \frac{1}{d_i^2}$.

   Hint Write a program to find the numbers for your.

   (d) (1 points) Prove $Q(8)$ is true by giving 8 numbers $d_1, \ldots, d_8$ such that $1 = \sum_{i=1}^{8} \frac{1}{d_i^2}$.

   Hint Write a program to find the numbers for your.

   (e) (6 points) Show that $(\forall n \ge 1)[Q(n) \rightarrow Q(n+3)]$.

   (We only need $n \ge 6$ to prove the theorem, but one can prove this true for $n \ge 1$.)

Do this problem on the next page and, if needed, the page after that.

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2. (10 points)

(a) (0 points but you will need this later) Show that

$$1 = \frac{1}{2} + \frac{1}{3} + \frac{1}{6}.$$

(b) (0 points but you will need this later) Show that

$$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}.$$

(c) (5 points)
Use the first and second parts of this problem to prove
The Reciprocal Theorem
For all $n \geq 3$ there exists $d_1 < \cdots < d_n$ such that $\sum_{i=1}^{n} \frac{1}{d_i} = 1$.

(d) (5 point) Use your proof and a computer program to write 1 as a sum of 10 reciprocals. Give us those 10. (You MUST use your proof.)

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3. (10 points- 5 each) Let L_1 and L_2 be two linear orderings.

   An **Order-Preserving bijection between** L_1 **and** L_2 is a bijection f with domain L_1, co-domain L_2 such that $x < y \rightarrow f(x) < f(y)$.

   For each of the following pairs of linear orderings say if there is an order preserving bijection between them.

   If YES then describe the order-preserving bijection.

   If NO then prove there is no order-preserving bijection.

   (a) $\mathsf{N} + \mathsf{Z}$ and $\mathsf{Z} + \mathsf{N}$.

   (b) $\mathsf{Z} + \mathsf{Z} + \mathsf{Z}$ and $\mathsf{Z} + \mathsf{Z}$.