

Honors Homework 7: Buddy can you spare a dime?

CMSC 250H

Due Date: Monday March 29, 9:00AM, NO DEAD CAT

What is the coefficient of x^{2021} in the Taylor Expansion of

$$\frac{1}{x^8 - x^7 - x + 1}.$$

Do by hand (NO programming) and show your work.

SOLUTION

$$\begin{aligned}\frac{1}{x^8 - x^7 - x + 1} &= \frac{1}{x - 1} \frac{1}{x^7 - 1} = \frac{1}{1 - x} \frac{1}{1 - x^7} \\ &= (1 + x + x^2 + x^3 + \cdots)(1 + x^7 + x^{14} + \cdots)\end{aligned}$$

The coefficient of x^n is the number of ways to make n cents with 1-coins and 7-coins. We call 1-cent coins **pennies** and 7-cent coins **emilies**.

Let

$f(n)$ be the number of ways to make n cents using pennies and emilies.

$$f(0) = 1$$

$$f(1) = f(2) = \cdots f(6) = 1.$$

$f(7) = 2$: either 7 pennies or 1 emily.

$f(8) = 2$: you NEED to use 1 penny. After that you have $f(7)$.

More generally, of $n \in \mathbb{N}$ and $0 \leq i \leq 6$, then

$$f(7n + i) = n + 1.$$

$$2021 = 7 * 288 + 5.$$

$$\text{So } f(2021) = 289.$$

So the answer is 289.

END OF SOLUTION