Honors HW 12. Due May 11

1. Let $n \in \mathsf{N}$.

   Alice has $a \in \{0,1\}^n$ on her forehead. Bob has $b \in \{0,1\}^n$ on her forehead. Carol has $c \in \{0,1\}^n$ on her forehead. Donna has $c \in \{0,1\}^n$ on her forehead.

   They view a, b, c, d as n-bits numbers.

   They want to know if $a + b + c + d = 2^n - 1$.

   Show how they can compute this with LESS THAN n bits of communication.

2. Let $n \in \mathsf{N}$. Let $i \in \mathsf{N}$. Think of $k \ll n$.

   Society now has done away with names and everyone is a number. A_1 has $a_1 \in \{0,1\}^n$ on her forehead. A_2 has $a_2 \in \{0,1\}^n$ on her forehead. $\ldots$ A_k has $a_k \in \{0,1\}^n$ on her forehead.

   They view $a_1, \ldots, a_k$ as n-bits numbers.

   They want to know if $a_1 + \cdots + a_k = 2^n - 1$.

   Show how they can compute this with LESS THAN n bits of communication.

   Recall that for the 2-egg problem we have that the number of drops needed is roughly $\sqrt{2}\sqrt{n}$.

   Let $D(e, n)$ be number of drops needed if you have e eggs and n floors.

3. Write a program that will, given e, n, compute $D(e', n')$ for all $1 \le e' \le e$ and $1 \le n' \le n$.

4. Run your program for $e = 3$ and $n = 1, \ldots, 100$. Graph the function. Try to determine what the function is approximately.

5. Run your program for $e = 4$ and $n = 1, \ldots, 100$. Graph the function. Try to determine what the function is approximately.

6. Is there some e such that

$$D(e, 100) = D(e+1, 100) = D(e+2, 100) \cdots ?$$