Honors HW 12. Due May 11

1. Let \( n \in \mathbb{N} \).

Alice has \( a \in \{0,1\}^n \) on her forehead. Bob has \( b \in \{0,1\}^n \) on her forehead. Carol has \( c \in \{0,1\}^n \) on her forehead. Donna has \( c \in \{0,1\}^n \) on her forehead.

They view \( a, b, c, d \) as \( n \)-bits numbers.

They want to know if \( a + b + c + d = 2^n - 1 \).

Show how they can compute this with LESS THAN \( n \) bits of communication.

2. Let \( n \in \mathbb{N} \). Let \( i \in \mathbb{N} \). Think of \( k \ll n \).

Society now has done away with names and everyone is a number. \( A_1 \) has \( a_1 \in \{0,1\}^n \) on her forehead. \( A_2 \) has \( a_2 \in \{0,1\}^n \) on her forehead. \( \ldots \) \( A_k \) has \( a_k \in \{0,1\}^n \) on her forehead.

They view \( a_1, \ldots, a_k \) as \( n \)-bits numbers.

They want to know if \( a_1 + \cdots + a_k = 2^n - 1 \).

Show how they can compute this with LESS THAN \( n \) bits of communication.

Recall that for the 2-egg problem we have that the number of drops needed is roughly \( \sqrt[2]{n} \).

Let \( D(e, n) \) be number of drops needed if you have \( e \) eggs and \( n \) floors.

3. Write a program that will, given \( e, n \), compute \( D(e', n') \) for all \( 1 \leq e' \leq e \) and \( 1 \leq n' \leq n \).

4. Run your program for \( e = 3 \) and \( n = 1, \ldots, 100 \). Graph the function.

Try to determine what the function is approximately.

5. Run your program for \( e = 4 \) and \( n = 1, \ldots, 100 \). Graph the function.

Try to determine what the function is approximately.

6. Is there some \( e \) such that

\[
D(e, 100) = D(e + 1, 100) = D(e + 2, 100) \cdots ?
\]