Honors Homework 4 (Solutions):
Grid Coloring

CMSC 250H

Due Date:

1. Find a n x m grid such that for all 4-colorings of n x m there is a mono rectangle.

   41 x 5, 31 x 6, 29 x 7, 25 x 9, 23 x 10, 22 x 11, 21 x 13, 19 x 17, 17 x 19, 13 x 21, 11 x 22, 10 x 23, 9 x 25, 7 x 29, 6 x 31, 5 x 41

2. For all c find n and m such that for all c-colorings of n x m there is a mono rectangle.

   Let us first look at this problem a row at a time. We have c colors that we want to color n dots. Let $n = c + 1$. By the Pigeonhole Principle, we know that each row will have at least 2 of the same color in it. Now lets choose our m. Let us look at how many ways make a row. Well, we have c colors and $c + 1$ spots. So, we have $(c \times c \times ... \times c)$ $c + 1$ times or c^{c+1} ways. So if $m = c^{c+1}$, we must have a monochromatic rectangle. (Note you can do much better then this but this is the most intuitive)

3. (0 points) Challenge Problem: There is an n such that for all 2-colorings of n x n there exists a mono SQUARE. Give me something even if you have no idea to show you thought about it.

   Will be presented by Dr. Gasarch.