

Homework 03, Morally due Mon Feb 22, 9:00AM

1. (0 points but if you miss the midterm that means you got this wrong retroactively and you will lose a lot of points). When is the midterm? By what day do you need to tell Dr. Gasarch that you cannot make the midterm (if you cannot know ahead of time)?

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2. (50 points) In the country of Fredonia they play a game that is similar to what we call Poker but with a different deck of cards.

Every card has a rank from $\{1, 2, \dots, R\}$.

Every card has a suite from $\{1, \dots, S\}$.

Every player gets C cards.

- (a) (15 points) (DO NOT allow wraparound.) What is prob of a straight that is NOT a flush. We DO NOT allow wrap-around, so if $R = 10$ and $C = 7$ then 10-1-2-3-4-5-6 does NOT counts.
- (b) (15 points) (DO allow wraparound.) What is prob of a straight that is NOT a flush. We DO NOT allow wrap-around, so if $R = 10$ and $C = 7$ then 10-1-2-3-4-5-6 does counts.
- (c) (10 points) Give a formula which is a function of R, S, C for the difference between

Prob of a straight that is NOT a flush, ALLOWING wraparound (Question b)
and

Prob of a straight that is NOT a flush NOT ALLOWING wraparound (Question a)
We call this formula $DIFF(R, S, C)$.

- (d) (0 points) Write a program that will, given R, S, C , compute $DIFF(R, S, C)$. (You should pre-compute binomial coefficients ahead of time rather than having to keep recomputing them.)

Send your code to Emily by email. Send the actual .java/.py/ect file. You need to use your .umd email address or it will not send. In your pdf, you must have the output your code provides. You can screenshot this or type it in.

Hint: Use Python.

- (e) (10 points) Run the program for all $1 \leq R, S, C \leq 10$. For which R, S, C is the diff between the two numbers greatest? Smallest? If you want to make the diff small, which variable should you change, and in what direction?

SOLUTION

Note that the total number of hands is $\binom{RS}{C}$.

- (a) A straight that is NOT a flush.

Pick a rank r to be the first card in the straight. Since the cards will be of ranks $r, r+1, \dots, r+C-1$ need to have $r+C-1 \leq R$ so $r \leq R-C+1$. So there are $R-C+1$ ways to pick r .

Then you have $r, \dots, r+C-1$. Now pick for each card a suite in one of S ways, but CANNOT have them all be of the same suite. We look at ALL ways to pick suits and then subtract those where they are all the same suite.

All ways: S^C .

Number of ways where its all the same suite: S .

So there are $S^C - S$ ways to pick the suite.

Hence the Prob that a hand is straight-non-flush (without wrap around) is

$$\frac{(R-C+1)(S^C - S)}{\binom{RS}{C}}$$

- (b) The only difference in the allow-wrap-around case is that any of the ranks can be the least rank. Hence the answer is

$$\frac{R(S^C - S)}{\binom{RS}{C}}$$

- (c) The difference between the wrap-around and not-wrap around cases is

$$\frac{R(S^C - S)}{\binom{RS}{C}} - \frac{(R-C+1)(S^C - S)}{\binom{RS}{C}}$$

Which is

$$\frac{S^C - S}{\binom{RS}{C}}(C+1)$$

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3. (50 points) We are again in Fredonia (see prior problem). However, we are assuming that C is even and $S/2 < C < S$.
- (a) (15 points) A *Full House* in Fredonia poker is when there are $C/2$ cards of the same rank, and $C/2$ cards of another rank. (Example: If $R = 10$, $C = 6$ and the suites are H,C,S,D then (2H, 2C, 2D, 8H, 8S, 8D) is a full house. What is the prob of getting a full house.
- (b) (15 points) Let $k \leq S$. A *k-of-a-kind* (henceforth *k-kind*) is a hand where there are k of the same rank. (Example: If $R = 10$, $C = 6$ the suites are H,C,S,D,Z,W and $k = 4$ then (3H, 3C, 3S, 3D, 9H, 10D) is a 4-kind. What is the prob of getting a k -kind. NOTE- we ALLOW to have the hand be even stronger. For example (3H, 3C, 3S, 3D, 9H, 9D) is fine, and even (3H, 3C, 3S, 3D, 3Z, 3W) is fine.
- (c) (10 points) Give a formula which is a function of R, S, C, k for the difference between
- Prob of a k -kind (Question b)
- and
- Prob of a full house (Question a)
- We call this formula $DIFF(R, S, C, k)$.
- (d) (0 points) Write a program that will, given R, S, C $S/2 < C < S$ and C even, compute $DIFF(R, S, C, k)$. (You should pre-compute binomial coefficients ahead of time rather than having to keep recomputing them.)
- Send your code to Emily by email. Send the actual .java/.py/ect file. You need to use your .umd email address or it will not send. In your pdf, you must have the output your code provides. You can screenshot this or type it in.
- Hint: Use Python.
- (e) (10 points) For each triple $4 \leq R, S, C \leq 10$ with $S/2 < C < S$ and C even, run the program with $k = C/2, C/2+1, \dots, S$, put it in a table, and answer the following: For which k is $DIFF(R, S, C, k)$ smallest (it can be negative)? largest? Closest to 0?

SOLUTION

Note that the total number of hands is still $\binom{RS}{C}$.

(a) We count the number of full houses. First pick 2 ranks, so

$$\binom{R}{2}$$

(In the above example we picked ranks (2,8).)

Then pick for the first rank the $C/2$ suits

$$\binom{S}{C/2}$$

(In the above example we picked suites (H,C,D).)

Then pick for the second rank the $C/2$ suits

$$\binom{S}{C/2}$$

(In the above example we picked suites (H,S,D). Would have been fine if it was also (H,C,D).)

So the prob of a full house is

$$\frac{\binom{R}{2} \binom{S}{C/2} \binom{S}{C/2}}{\binom{RS}{C}}.$$

(b) First pick the rank, one of R ways

(In the above example we picked rank 3.)

Then pick k suites in $\binom{S}{k}$ ways.

(In the above example we picked suites (H,C,S,D).)

You have picked k cards out of RS cards, so need to pick $C - k$ cards out of the remaining $RS - k$ cards. This can be done

$$\binom{RS - k}{C - k}$$

ways.

So the prob of a k -kind is

$$\frac{R \binom{S}{k} \binom{RS-k}{C-k}}{\binom{RS}{C}}.$$

(c) Omitted

(d) Omitted