Homework 04, Morally due Mon Mar 1, 9:00AM

For Programming Problems: Send your code to Emily by email. Send the actual .java/.py/ect file. You need to use your .umd email address or it will not send. In your pdf, you must have the output your code provides. You can screenshot this or type it in. Hint: Use Python.

1. (0 points but if you miss the midterm that means you got this wrong retroactively and you will lose a lot of points). When is the midterm? By what day do you need to tell Dr. Gasarch that you cannot make the midterm (if you cannot and know ahead of time)?

2. (20 points) Emily tosses m balls into n boxes at random. Assume $m \ll n$.

   (a) (10 points) What is the probability that at least FOUR balls are in the same box. (You may use the approximations we used for the problem of THREE balls.)

   (b) (10 points) Let $k \ll n$. What is the probability that at least k balls are in the same box. (You may use the approximations we used for the problem of THREE balls.)

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3. (30 points) In Fredonia they use 4-sided dice! And the dice often have unequal probabilities.

   Assume we have dice such that prob of getting an i is p_i. Note that
   For all i, $0 \leq p_i \leq 1$ and $p_1 + p_2 + p_3 + p_4 = 1$.

   (a) (6 points) Emily roles two dice. For $2 \leq n \leq 8$ let S_n be the prob that the sum is n. For $2 \leq n \leq 8$ give a formula for S_n in terms of p_1, p_2, p_3, p_4.

   (b) (0 points) Write a program that will, given p_1, p_2, p_3, p_4 do the following:

   i. Compute $S_2, \ldots, S_8$
   ii. Compute the MAX of $S_2, \ldots, S_8$, the MIN of $S_2, \ldots, S_8$, and MAX-MIN.
   iii. Compute the diff of prob of S_2 and prob of S_4.
   iv. Compute the diff of prob of S_8 and prob of S_4.

   (c) (0 points) Run your program on all tuples of (p_1, p_2, p_3, p_4) such that

   - $p_1, p_2, p_3, p_4 \in \{\frac{1}{10}, \ldots, \frac{9}{10}\}$.
   - $p_1 + p_2 + p_3 + p_4 = 1$.

   (d) (6 points) For which (p_1, p_2, p_3, p_4) is the DIFF between MAX and MIN the smallest? The largest?

   (e) (6 points) For which (p_1, p_2, p_3, p_4) is Prob of S_2 GREATER than the Prob of S_4.

   (f) (6 points) For which (p_1, p_2, p_3, p_4) is (1) Prob of S_2 GREATER than the Prob of S_4 AND (2) Prob of S_8 GREATER than the Prob of S_4.

   (g) (6 points) How many (p_1, p_2, p_3, p_4) did you have to consider? For the mundane question of points on the HW YES you can just give a number, perhaps by using your program. However, there IS a way to do it as a combinatorics problem, so try to figure it out as well.

   (h) (0 points) Speculate: Do you think there is SOME (p_1, p_2, p_3, p_4) with $0 \leq p_i \leq 1$ and $p_1 + p_2 + p_3 + p_4 = 1$ such that DIFF=0, so ALL of the sums are equally likely?

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4. (20 points) Let $\epsilon > 0$ and $n \in \mathsf{N}$. Think of ϵ as being very small.

   There are two coins

   Coin F (for Fair) has prob of Heads = Prob of tails = $\frac{1}{2}$.

   Coin B (for Bias) has prob of Heads = $\frac{1}{2} + \epsilon$ and prob of tails = $\frac{1}{2} - \epsilon$.

   Bill choose a coin at random (prob 1/2-1/2) and flips it n times. He gets all H.

   (a) (10 points) What is the prob that the coin is biased? (this will be a function of n and ϵ).

   (b) (0 points) Write a program that, on input ϵ, n prints the prob that the coin is biased.

   (c) (0 points) Write a program that, given ϵ, prints the smallest value of n so that if there are n H in a row the prob of bias is > 0.9.

   (d) (10 points) Run your program on the 7 values:
   $\epsilon = \frac{1}{3}, \frac{1}{3^2}, \ldots, \frac{1}{3^7}$.
   Is the value of n getting bigger? smaller? how fast?

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5. (10 point) Bill has a list of 528 celebrity birthdays born between Jan 1, 1925 and Dec 31, 1942. (None were born on Feb 29 so you can ignore leap days.)

   You may use formulas from the slides on the birthday paradox for this problem.

   (a) (5 points) What is the probability that there exists at least 3 of them that were born the same day AND year? (assume there births are randomly distributed).

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(b) (5 points)What is the probability that there exists at least 4 of them that were born the same day AND year? (assume there births are randomly distributed).