Homework 05, Morally due Mon Mar 22, 9:00AM

For Programming Problems: Send your code to Emily by email. Send the actual .java/.py/ect file. You need to use your .umd email address or it will not send. In your pdf, you must have the output your code provides. You can screenshot this or type it in. Hint: Use Python.

1. (0 points but if you miss the midterm that means you got this wrong retroactively and you will lose a lot of points). When is the SECOND midterm? By what day do you need to tell Dr. Gasarch that you cannot make the midterm (if you cannot and know ahead of time)?

2. (10 points) Write an expression with quantifiers that means there are NO elements between x and y. Call it NOBET for later problems.

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3. (15 points) For each sentence below give

   (1) a domain $D \subseteq \mathsf{R}$ such that over that domain the statement is TRUE, and

   (2) a domain $D \subseteq \mathsf{R}$ such that over that domain the statement is FALSE.

   Explain why your answer works.

   (a) (5 points) $(\forall x)(\exists y)[x < y \wedge \text{NOBET}(x, y)]$

   (b) (10 points) $(\exists x)[(\forall y \neq x)[y < x \wedge (\exists z)[y < z \wedge \text{NOBET}(y, z)]]]$

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4. (15 points)

   (a) (0 points) Write a program that will, given a number, determine the LEAST number of SQUARES that add up to it and output a way to do that. For example:

   On input 6 it outputs

   $6 = 2^2 + 1^2 + 1^2$, so 3.

   (b) (10 points) Run your program on $1, \ldots, 1000$. Give both ALL of your data and also the numbers that needed 4 squares.

   (c) (5 points) Make a conjecture of the form:

   If x satisfies property BLAH then x requires 4 squares.

   Every number between 1 and 1000 that has property BLAH has to require 4 squares (though we do not insist on the converse).

   There must be an infinite number of numbers that satisfy BLAH.

   (EXAMPLE which is not true: BLAH is that x is the product of a prime and a square.)

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5. (15 points)

   (a) (0 points) Write a program that will, given a number, determine the LEAST number of CUBES that add up to it and output a way to do that. For example:

   On input 15 it outputs

   $15 = 2^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3 + 1^3$ so 8.

   (b) (10 points) Run your program on $1, \ldots, 1000$. Let MAX be the MAX number of cubes that a number needed.

   For all $1 \leq i \leq MAX$ state how many numbers needed i cubes.

   (c) (5 points) Make a conjecture of the form:

   For all sufficiently large n, n can be written as the sum of X cubes.

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6. (15 points) We are working in binary. When we input 3 bits to a circuit we think of it as a NUMBER in base 2.

   000=0

   001=1

   010=2

   011=3

   100=4

   101=5

   110=6

   111=7

   The output will be 1 bits.

   (a) (10 points) Write a truth table with 3 inputs and 1 outputs for the following function:

$$f(xyz) = \begin{cases} 1 & \text{if } xyz \text{ is prime;} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

   (0 and 1 are not primes.)

   So for example

   $f(111) = 1$ since 7 is prime.

   (b) (5 points) Using the truth table write a formula for the function f. DO NOT SIMPLIFY as it will make it harder for Emily to grade.

   (c) (0 points and don't turn in) Draw a circuit for the function f.

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7. (15 points)

   (a) (5 points) Show that there exists a Boolean formula on 10 variables such that IF you did the truth table there would be exactly 239 rows that return TRUE.

   (b) (10 points) For which k, n does there exist a boolean Formula on n variables such that IF you did the truth table there would be exactly k rows that return TRUE.

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8. (15 points) Let $\phi(x_1, \ldots, x_n)$ and $\psi(x_1, \ldots, x_n)$ be Boolean formulas on n variables. They are *equivalent* if they have the same truth table.

   We want to count the number of boolean formula but NOT want to count ϕ and ψ if they are equivalent.

   How big is the largest set of non-equivalent Boolean Formulas on n variables.