CMSC 250 Second Midterm

1. This is an open-everything exam. You can use anything except ask another person. Caution: if you copy from the web or elsewhere mindlessly you will probably get it wrong.

2. There are 3 problems which add up to 70 points. Recall that you already did 30 points of this midterm take home.

3. The exam is April 8 from 8:00PM until 10:15PM unless you have contacted me to make other arrangements. So the exam is 2 hours and 15 minutes.

4. For each question show all of your work and use \LaTeX{} or write VERY NEATLY. Clearly indicate your answers. No credit for illegible answers.

5. Please write out the following statement: I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.
1. (25 points) Prove that $7^{1/3}$ is NOT rational. You must state and prove carefully any lemmas you use.

You can do this problem on this page and the next page.
2. (25 points) Let $a_n$ be defined as follows.

$$a_1 = 11$$

$$(\forall n \geq 2)[a_n = a_{\lfloor n^{3/4} \rfloor} + a_{\lfloor n^{1/4} \rfloor} + 15]$$

Show by strong induction that

$$(\forall n \geq 1)[a_n \equiv 11 \pmod{13}]$$

Include Base Case, IH, and IS.

You can do this problem on this page and the next page.
3. (20 points) Note that \( \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1 \).

Also note that \( \frac{1}{2} + \frac{1}{3} + \frac{1}{7} + \frac{1}{42} = 1 \).

We obtained the 2nd equation from the 1st by noticing that \( \frac{1}{6} = \frac{1}{7} + \frac{1}{42} \).

This equation is a special case of the equation \( \frac{1}{d} = \frac{1}{d+1} + \frac{1}{d(d+1)} \)

which can be proven algebraically (it does not need a proof by induction).

OKAY, now finally the problem.

Show that, for all \( n \geq 3 \), there exists natural numbers \( d_1 < \cdots < d_n \) such that

\[ \frac{1}{d_1} + \cdots + \frac{1}{d_n} = 1. \]

(HINT Do this by induction on \( n \) and use

\[ \frac{1}{d} = \frac{1}{d+1} + \frac{1}{d(d+1)}. \])