The set of bijections from N to N is uncountable

250H

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Let $g(0) \in E - f_1(0)$ and
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There is a one to one function from the even number to the even numbers.

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g does not appear in our original list of bijections as $g(2n) \neq f_n(2n)$. This is a contradiction. So there is no bijection from $\mathbb{N}$ to B.