Ferrers Diagrams

250H
Integer Partition

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Def: A partition of a positive integer $n$, also called an integer partition, is a way of writing $n$ as a sum of positive integers.

How many ways can we partition 4? (Order does not matter, ie $1+2 = 2+1$)

- 4
- $3 + 1$
- $2 + 2$
- $2 + 1 + 1$
- $1 + 1 + 1 + 1$
Integer Partition

Def: $p(n)$ is defined as the number of partitions of $n$
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What is $p(4)$?
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What is $p(4)$? 5

- 4
- 3 + 1
- 2 + 2
- 2 + 1 + 1
- 1 + 1 + 1 + 1
Integer Partition

Def: \( p(n) \) is defined as the number of partitions of \( n \)

What is \( p(4) \)? 5

- 4
- 3 + 1
- 2 + 2
- 2 + 1 + 1
- 1 + 1 + 1 + 1

What is \( p(0) \)?
Integer Partition

Def: $p(n)$ is defined as the number of partitions of $n$

What is $p(4)$? 5

- $4$
- $3 + 1$
- $2 + 2$
- $2 + 1 + 1$
- $1 + 1 + 1 + 1$

What is $p(0)$? 1
Notation

Math people are lazy
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\[ 2 + 2 + 1 = (2, 2, 1) = (2^2, 1) \]
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\[ 2 + 2 + 1 = (2, 2, 1) = (2^2, 1) \]

● Note: \( 2^2 \) does not mean 2 squared.
  ○ Instead it means how many times that number appears in a partition.
  ○ Don’t use this because its confusing, but if you see it out in the wild this is what it means.
We only want to look at Pennies and Nickels.

Let $p(0) = p(1) = p(2) = p(3) = p(4) = 1$. Let $n \geq 5$.

$p(n)$: use a nickel or don’t

- If we use a nickel then $p(n-5)$
  - makes sense since $n \geq 5$
- If we do not use nickels then you have $n$ cents, only pennies, so 1 way
  - $p(n) = p(n-5)+1$
Everyone's Favorite Forced Social Time

- In breakout rooms, try and find a pattern for large n’s
Pattern

if $0 \leq i \leq 4$:

- $p(5n+i)$ we have to use $i$ pennies, so this is $p(5n)$
- $p(5n) = p(5(n-1)) + 1$
- $p(5n+i) = n+1$
Another Answer

Coefficients of $x^n$ in:

\[
\frac{1}{1-x} \left( \frac{1}{1-x^5} \right) = \frac{1}{(1-x)(1-x^5)} = \frac{1}{x^6 - x^5 - x + 1}
\]

\[
\frac{1}{x^6 - x^5 - x + 1} = 1 + x + x^2 + x^3 + x^4 + 2x^5 + 2x^6 + 2x^7 + 2x^8 + 2x^9 + 3x^{10} + 3x^{11} + \ldots
\]

This is an odd approach to finding taylor series: If we want to find the taylor series of a function, first find a change problem that it answers, solve that change problem, and you have the taylor series.
Partitions

Partitions can be viewed graphically with Ferrer Diagrams
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Let’s look at 4’s Ferrer Diagrams.
How to make a Ferrer Diagram

- Finite collection of boxes or dots
- Arranged in Left-Justified Rows
- Row Lengths in non-increasing order
Notation

Young Diagram

Ferrers Diagram
That's dumb they should be called the same thing.
Notation

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For our purposes yes, but each actually has a different purpose in the greater scope of math
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Young Diagrams are useful in

- Symmetric functions and group representation theory
- Polyominoes
Notation again

Why does this have so many notations for such a simple thing? IDK
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English Notation

French Notation
Notation again

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English Notation

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Yes this one is dumb. Mathematicians get heated about this.
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Yes this one is dumb. Mathematicians get heated about this.

In his book “symmetric functions”, Macdonald tells readers preferring the French convention to "read this book upside down in a mirror" (Macdonald 1979, p. 2)
Theorem

- The number of ways to partition \( n \) into \( < m \) parts is the number of ways to partition \( n \) into parts the largest of which is \( < m \).
Theorem

- The number of ways to partition $n$ into $< m$ parts is the number of ways to partition $n$ into parts the largest of which is $< m$
- The number of ways of partitioning $n$ into $m$ parts is equal to the number of ways of partitioning $n$ into parts, the largest of which is $m$. 
Let’s look at 10 with $n = 3$

$4 + 3 + 3$

$1 + 3 + 3 + 3$
Let’s look at 10 with \( n = 3 \)

\[
4 + 3 + 3 \quad \text{and} \quad 1 + 3 + 3 + 3
\]

What is \( m \) in both of these diagrams?
Let’s look at 10 with $n = 3$

What is $m$ in both of these diagrams? 3
Theorems

Lemma: The number of partitions of $n$ with no parts equal to 1 is $p(n) - p(n-1)$
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Theorem: The number of partitions of $n$ into distinct parts equals the number of partitions of $n$ into odd parts.
Theorems

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Theorem: The number of partitions of $n$ into distinct parts equals the number of partitions of $n$ into odd parts.

Theorem: The number of partitions of $n$ into parts that are both odd and distinct is equal to the number of self-conjugate partitions of $n$
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Theorem: The number of partitions of $n$ into distinct parts equals the number of partitions of $n$ into odd parts.

Theorem: The number of partitions of $n$ into parts that are both odd and distinct is equal to the number of self-conjugate partitions of $n$. (This can be proved using Ferrer diagrams)