True or False?
Is the following TRUE or FALSE:

\[(\forall x)(\forall y)\left[ x < y \rightarrow \left( \exists z \right) \left[ x < z < y \right] \right] \]
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**Answer** This is a stupid question! Need to specify the Domain.
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**Better Questions** Let $D$ mean Domain.

1) If $D = \mathbb{N}$ then is the statement true?
True of False: Density

Is the following TRUE or FALSE:

\[(\forall x)(\forall y)[x < y \rightarrow (\exists z)[x < z < y]]\]

**Answer** This is a stupid question! Need to specify the Domain.

**Better Questions** Let \(D\) mean Domain.

1) If \(D = \mathbb{N}\) then is the statement true? No. Counterexample: \(x = 1, y = 2\). There is no element \(z \in \mathbb{N}\) such that \(1 < z < 2\).
True of False: Density

Is the following TRUE or FALSE:

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**Answer** This is a stupid question! Need to specify the Domain.

**Better Questions** Let $D$ mean Domain.

1) If $D = \mathbb{N}$ then is the statement true? No. Counterexample: $x = 1, y = 2$. There is no element $z \in \mathbb{N}$ such that $1 < z < 2$.

2) If $D = \mathbb{Q}$ then is the statement true?
True of False: Density

Is the following TRUE or FALSE:

\[(\forall x)(\forall y)[x < y \rightarrow (\exists z)[x < z < y]]\]

**Answer** This is a stupid question! Need to specify the Domain.

**Better Questions** Let \(D\) mean Domain.

1) If \(D = \mathbb{N}\) then is the statement true? No. Counterexample: \(x = 1, y = 2\). There is no element \(z \in \mathbb{N}\) such that \(1 < z < 2\).

2) If \(D = \mathbb{Q}\) then is the statement is true? Yes. Take \(z = \frac{x+y}{2}\).
Find Domains such that . . .

Consider:

$$(\exists x)(\forall y \neq x)[y > x]$$
Find Domains such that . . .

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Consider:

$$(\exists x)(\forall y \neq x)[y > x]$$

Give a domain where this is T. $\mathbb{N}$ with $x = 0$.
Give a domain where this is F. $\mathbb{Z}$ since, forall $x$, $x - 1 < x$. 
Expressing Math With Quantifiers
I want to say $x$ is even. How to do that with quantifiers.
I want to say $x$ is even. How to do that with quantifiers. Quantifiers range over $\mathbb{Z}$.

$$\text{EVEN}(x) \equiv (\exists y)[x = 2y]$$
I want to say that $x \equiv 1 \pmod{5}$, which means that when we divide $x$ by 5 we get a remainder of 1. Let's call this property $\text{ONEFIVE}$.
Expressing Properties of Numbers: $\equiv 1 \pmod{5}$

I want to say that $x \equiv 1 \pmod{5}$, which means that when we divide $x$ by 5 we get a remainder of 1. Let's call this property ONEFIVE. Quantifiers range over $\mathbb{Z}$.

$$\text{ONEFIVE}(x) \equiv (\exists y)[x = 5y + 1]$$
I want to say that $x \in \mathbb{N}$ is PRIME.
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PRIMES over $\mathbb{N}$
I want to say that $x \in \mathbb{N}$ is PRIME.
Quantifiers range over $\mathbb{N}$.

$$\text{PRIME}(x) \equiv (x \neq 0, 1) \land (\forall y, z)[x = yz \rightarrow (y = 1) \lor (z = 1)]$$
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Does this work? Discuss.
I want to say that $x \in \mathbb{Z}$ is PRIME. Quantifiers range over $\mathbb{Z}$.

$$\text{PRIME}(x) \equiv (x \neq 0, 1) \land (\forall y, z)[x = yz \rightarrow (y = 1) \lor (z = 1)]$$

Does this work? Discuss.

$-7 = -1 \times 7$ Its also $-7 \times -1 \times -1 \times 1$. So... not a prime?
I want to say that $x \in \mathbb{Z}$ is PRIME. Quantifiers range over $\mathbb{Z}$.

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Does this work? Discuss.

$-7 = -1 \times 7$ Its also $-7 \times -1 \times -1 \times 1$. So... not a prime? NAH, we want $-7$ to be a prime.
PRIME \( x \) \( \equiv (x \neq 0, 1) \land (\forall y, z)[x = yz \rightarrow (y = 1) \lor (z = 1)] \)
PRIMES over \( \mathbb{Z} \) (cont)

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\text{PRIME}(x) \equiv (x \neq 0, 1) \land (\forall y, z)[x = yz \rightarrow (y = 1) \lor (z = 1)]
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Why did we make 1 an exception? Because \( 7 = 1 \times 7 \).
PRIME$(x) \equiv (x \neq 0, 1) \land (\forall y, z)[x = yz \rightarrow (y = 1) \lor (z = 1)]$

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\text{PRIME}(x) \equiv (x \neq 0, 1, -1) \land (\forall y, z)[x = yz \rightarrow (y = \pm 1) \lor (z = \pm 1)]
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Def The Gaussian Integers $G$ are numbers of the form

$$\{a + bi : a, b \in \mathbb{Z}\}$$
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We want to define PRIME in \( G \). What will be the exceptional numbers? Why?

Breakout Rooms!
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**Breakout Rooms!**

The exceptions are $\{1, -1, i, -i\}$. Why?
Def The **Gaussian Integers** $G$ are numbers of the form

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The exceptions are $\{1, -1, i, -i\}$. Why?

$7 = i \times -i \times 7$.

We don’t really want to count the $i$ and $-i$. 
Def Let $D$ be some domain. If $x \in D$ then the mult inverse of $x$ (if it exists) is the number $y$ such that $xy = 1$. 

In $\mathbb{N}$ the only number that has a mult inverse is 1.

In $\mathbb{Z}$ the only numbers that has a mult inverses are 1, $-1$.

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Def Let $D$ be a domain. The units of $D$ are the elements of $D$ that have a multiplicative inverse. The Unit are the exceptions. If $x \in D$, $u$ is a unit, and $v$ is its inverse, then $x = uvx$.

We don't want to say $x$ is not prime. $u$, $v$ should not matter!
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Units and Primes

Let $D$ be any domain of numbers. We will be quantifying over it.

$\text{UNIT}(x) \equiv (\exists y)[xy = 1]$

$\text{PRIME}(x) \equiv (x \neq 0, x \not\in \text{UNIT}) \land (\forall y, z)(x = yz \rightarrow ((y \in \text{UNIT}) \lor (z \in \text{UNIT})))$.
Units and Primes

Let $D$ be any domain of numbers. We will be quantifying over it.

$$\text{UNIT}(x) \equiv (\exists y)[xy = 1]$$

$$\text{PRIME}(x) \equiv$$

$$(x \neq 0, x \notin \text{UNIT}) \land (\forall y, z)[x = yz \rightarrow ((y \in \text{UNIT}) \lor (z \in \text{UNIT})].$$
1) So thats why 1 is NOT a prime. In any domain $D$ we have **Units, Primes, Composites, 0**
1) So thats why 1 is NOT a prime. In any domain \( D \) we have 
**Units, Primes, Composites, 0**

2) Can we define primes in \( \mathbb{Q} \)?
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2) Can we define primes in $\mathbb{Q}$? Discuss
So Thats why...

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All elements of $\mathbb{Q}$ are units, so there are no primes.
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All elements of $\mathbb{Q}$ are units, so there are no primes.

3) Let $\text{ONEFOUR} = \{n : n \equiv 1 \pmod{4}\}$. The only unit is 1.
What are the primes in $\text{ONEFOUR}$?
So Thats why... 

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Primes in ONEFOUR

Elements of ONEFOUR: 1, 5, 9, 13, 17, 21, 25. We stop here.
1: a unit
5: a prime
9: a prime! Note that 3 ∉ ONEFOUR so cannot say 9 = 3 × 3.
13, 17: Primes
21: a prime!
25: 5 × 5 are first composite.
Expressing Theorems: Four-Square Theorem

**Four-Square Theorem** Every natural number is the sum of \( \leq 4 \) squares.
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\[
(\forall x)(\exists x_1, x_2, x_3, x_4) [x = x_1^2 + x_2^2 + x_3^2 + x_4^2]
\]
Goldbach’s Conjecture  Every sufficiently large even number can be written as the sum of two primes.
Expressing Statements: Goldbach’s Conjecture

Goldbach’s Conjecture  Every sufficiently large even number can be written as the sum of two primes.

\[ \exists x (\forall y > x) \]

\[
[\text{EVEN}(y) \rightarrow (\exists y_1, y_2)[\text{PRIME}(y_1) \land \text{PRIME}(y_2) \land y = y_1 + y_2]]
\]
Vinogradov’s Theorem

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\((\exists x)(\forall y > x)\)

\([\text{ODD}(y) \rightarrow (\exists y_1, y_2, y_3)[\text{PRIME}(y_1) \land \text{PRIME}(y_2) \land \text{PRIME}(y_3) \land y = y_1 + y_2 + y_3]\]

...
Square root of 2

We want to express this with quantifiers over \( \mathbb{Z} \). Note that if \( 2 = x^2 y^2 \) then \( 2y^2 = x^2 \).

\[
\neg \exists (x, y) [2y^2 = x^2] \iff \forall (x, y) \neg [2y^2 = x^2]
\]

Note that using \( \neg \exists (x, y) \equiv \forall (x, y) \neg \) ended up not having a \( \neg \) in the final expression.
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**Thm** $\sqrt{2} \notin \mathbb{Q}$. (We will prove this later in the course.)
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Note that if $2 = \frac{x^2}{y^2}$ then $2y^2 = x^2$. 
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Order Notation
Sometimes We Don’t Care About Constants

The following conversation would never happen.
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The following conversation would never happen.

**EMILY:** Bill, I have an algorithm that solves SAT in roughly $n^2$ time!

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**EMILY:** There are constants $c, d, e$ such that my algorithm works in time $\leq cn^2 + dn + e$. OH, the algorithm only has this runtime when the number of variables is $\geq 100$. 
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**BILL:** What are $c, d, e$?

**EMILY:** Who freakin cares! I solved SAT without using brute force and you are concerned with the constants!
When Do/Don’t We Care About Constants?

1) When we first look at a problem we want to just get a sense of how hard it is:
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When Do/Don’t We Care About Constants?

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If roughly $n^2$ then can we get it to roughly $n \log n$ or $n$?
1) When we first look at a problem we want to just get a sense of how hard it is:
   Exp vs Poly time?
   If poly then what degree?
   If roughly $n^2$ then can we get it to roughly $n \log n$ or $n$?
   Once we have exhausted all of our tricks to get it into (say) roughly $n^2$ time we THEN would do things to get the constant down, perhaps non-rigorous things.
We Want to Make “Roughly” Rigorous

We want to say that we don’t care about constants.
We Want to Make “Roughly” Rigorous

We want to say that we don’t care about constants. We want to say that $18n^3 + 8n^2 + 12n + 1000$ is roughly $n^2$. 

We leave it to the reader to prove that $18n^3 + 8n^2 + 12n + 1000 = O(n^2)$ by finding the values of $c$, $d$. 
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$f \leq O(n^2)$ First attempt:

$$(\exists c)[f(n) \leq cn^2].$$
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$f \leq O(n^2)$ Second and final attempt:
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by finding the values of $c, d$. 
\( f = O(g) \)

\( f \leq O(g) \) means

\[
(\exists n_0)(\exists c)(\forall n \geq n_0) [f(n) \leq cg(n)].
\]
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You will see \( O() \) a lot in CMSC 351 and 451 when you deal with algorithms and want to bound the run time, roughly.
Other Ways to Use $O()$

$f \in n^{O(1)}$ means poly time.
Other Ways to Use $O()$

$f \in n^{O(1)}$ means poly time.

$f \in 2^{O(n)}$ means $2^{cn}$ for some $c$, and after some $n_0$. 
The following conversation would never happen.
Another Conversations

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BILL: Emily, I have shown that SAT requires roughly $2^n$ time!
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EMILY: What are $c, d, e$?
BILL: Who freakin cares! I showed SAT is not in poly time you are concerned with the constants!
\( f = \Omega(g) \)

\( f \geq \Omega(g) \) means

\[(\exists n_0)(\exists c)(\forall n \geq n_0)[f(n) \geq cg(n)].\]
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\[
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\]

This notation is used to express that an algorithm requires some amount of time.
If I proved that SAT requires $\Omega(n^3)$ time would I have solved P vs NP?

No. SAT could still be in time $n^4$.

If I proved that SAT requires $n^{\Omega(\log \log \log n)}$ time would I have solved P vs NP?

Yes. That function is bigger than any poly. But result would be odd since people REALLY think SAT requires $2^{\Omega(n)}$.

You would still get the $1,000,000.$
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If I proved that SAT requires $\Omega(n^3)$ time would I have solved P vs NP?

No. SAT could still be in time $n^4$.

If I proved that SAT requires $n^{\Omega(\log \log \log n)}$ time would I have solved P vs NP?

Yes. That function is bigger than any poly. But result would be odd since people REALLY think SAT requires $2^{\Omega(n)}$.

You would still get the $1,000,000.$