MiniMax and Alpha Beta Pruning

CMSC 250H
Combinatorial Search

- Search algorithms that solve a particular problem by using large solution spaces
  - A* Search
  - Minimax
  - Alpha Beta pruning
- At each step, the algorithm looks at all possible combinations of decisions
Game Tree

MAX (x)

MIN (o)

MAX (x)

MIN (o)

TERMINAL

Utility -1 0 +1
Tic Tac Toe

- How many ways can you make the first move?
  - 9
- How many ways can a game of Tic-Tac-Toe be played?
  - 255,168
- The game tree will have 255,168 leaves
MiniMax

- Algorithm used in AI, Decision Theory, Game Theory, Stats, and Philosophy
  - Combinatorial Game Theory: Gives Game Solutions
- Idea: Minimize Loss in Worst Case
- Uses Recursion or Backtracking to make a Perfect Choice
- Slow!
  - Needs to visit every node
MiniMax
MiniMax
MiniMax

Max

Min

Max

Min

4

-4

0

3

4

-9

3
MiniMax

Max

Min

Max

Min
MiniMax
Bigger Example
Bigger Example
Tree Traversal

Pre-Order: Left Side of Bubble
{1, 2, 4, 5, 3, 6, 7}

In-Order: Bottom of Bubble
{4, 2, 5, 1, 6, 3, 7}

Post-Order: Right Side of Bubble
{4, 5, 2, 6, 7, 3, 1}
Alpha Beta Pruning

- Makes MiniMax more efficient
- If we search down the whole tree, the number of states is exponential to the depth of the tree
- Alpha Beta Pruning cuts away leaves when traversing tree
- Stops evaluating a state when at least one possibility has been found to prove worse than a previous found move
- Returns the same value that MiniMax would produce
- Prunes away branches that do not influence final decision
- In the tuple \([\alpha, \beta]\)
  - Maximize \(\alpha\)
  - Minimize \(\beta\)
Alpha Beta Pruning
Alpha Beta Pruning
Alpha Beta Pruning

(c) [3, +∞]

[3, 3]

3 12 8
Alpha Beta Pruning
Alpha Beta Pruning

(e)
Alpha Beta Pruning
Bigger Example
Bigger Example
maxValue(state, α, β)
    If (Terminal State)
        Return value
    Else
        For each child
            If (Player 2’s turn)
                α = max(α, minValue(state, α, β))
                If (α ≥ β)
                    return β
            Else
                β = min(β, maxValue(state, α, β))
        Return α
    Return α

minValue(state, α, β)
    If (Terminal State)
        Return value
    Else
        For each child
            If (Player 1’s turn)
                β = min(β, maxValue(state, α, β))
                If (β ≤ α)
                    return α
            Else
                α = max(α, minValue(state, α, β))
        Return β
    Return α
MiniMax vs. Alpha Beta Pruning Runtime

- **MiniMax**
  - Runtime: $O(b^n)$
  - Space: $O(bh)$

- **Alpha Beta Pruning**
  - Runtime:
    - Worst-Case: $O(b^n)$
    - Best-Case: $O(b^{h/2})$
  - Space: $O(bh)$

Why is the Worst-Case Runtime equal to MiniMax?

$b = \text{Branching Factor}$
$h = \text{Height of the Tree}$
MiniMax vs. Alpha Beta Pruning Runtime

- **MiniMax**
  - Runtime: $O(b^h)$
  - Space: $O(bh)$

- **Alpha Beta Pruning**
  - Runtime:
    - Worst-Case: $O(b^h)$
    - Best-Case: $O(b^{h/2})$
  - Space: $O(bh)$

Why is the Worst-Case Runtime equal to MiniMax?

In the Worst-Case, your Alpha Beta is running MiniMax!

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$b =$ Branching Factor  
$h =$ Height of the Tree
Alpha Beta for 2 Player Games

- Game Trees get really big really fast
  - Grows exponentially
  - Alpha Beta Pruning is more efficient than Minimax
- Used for many games
  - Tic-Tac-Toe
  - Chess
  - Go
- Heuristic is easily incorporated
  - A Heuristic is a mapping from a game state to a value
    - Ex: In Chess, White Pieces - Black Pieces = Value
      - This is a bad heuristic to use
  - We use heuristics when we do not want calculate every end game state
Real Life Use: Pokemon

- I created an AI simulation that simulates a competitive battling scenario
  - Used Java
  - Dictionary of Pokemon
  - Dictionary of Moves
  - Battle Game Tree
  - Alpha Beta Pruning to Traverse tree
  - Minimax to Check Alpha Beta
  - 12 different classes
**Example**

- **Play Rough**
- **Shadow Claw**
- **Switch to Marshadow**

**Mimikyu HP: 55**
**Deoxys HP: 50**

**Mimikyu HP: 55**
**Deoxys HP: 0**

- **Psycho Boost**
- **Ice Beam**
- **Switch**

**Mimikyu HP: 55**
**Deoxys HP: 35**

- **Psycho Boost**
  - **Ice Beam**
  - **Switch**

**Marshadow HP: 90**
**Deoxys HP: 50**

- **Psycho Boost**
- **Ice Beam**
- **Switch**

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Deoxys HP: 0

Mimikyu HP: 55
Deoxys HP: 35

Marshadow HP: 90
Deoxys HP: 50
Example

Play Rough

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Switch to Marshadow

Psycho Boost

Psycho Boost

Switch

Switch

Psycho Boost

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