Intro to Combinatorics
(“that n choose 2 stuff”)

CMSC 250
Jason’s sandwich
Jason’s Sandwich

- Suppose that Jason has the following ingredients to make a sandwich with:
  - White or black bread
  - Butter, Mayo or Honey Mustard
  - Romaine Lettuce, Spinach, Kale
  - Bologna, Ham or Turkey
  - Tomato or egg slices
Jason’s Sandwich

- Suppose that Jason has the following ingredients to make a sandwich with:
  - White or black bread
  - Butter, Mayo or Honey Mustard
  - Romaine Lettuce, Spinach, Kale
  - Bologna, Ham or Turkey
  - Tomato or egg slices

- **How many different sandwiches can Jason make?**
Jason’s Sandwich

• Suppose that Jason has the following ingredients to make a sandwich with:
  • White or black bread 2 options
  • Butter, Mayo or Honey Mustard 3 options
  • Romaine Lettuce, Spinach, Kale 3 options
  • Bologna, Ham or Turkey 3 options
  • Tomato or egg slices 2 options

• How many different sandwiches can Jason make?
  • $2 \times 3 \times 3 \times 3 \times 2 = 4 \times 27 = 108$
The Multiplication Rule

• Suppose that $E$ is some experiment that is conducted through $k$ sequential steps $s_1, s_2, \ldots, s_k$, where every $s_i$ can be conducted in $n_i$ different ways.
The Multiplication Rule

• Suppose that \( E \) is some experiment that is conducted through \( k \) sequential steps \( s_1, s_2, \ldots, s_k \), where every \( s_i \) can be conducted in \( n_i \) different ways.
  • Example: \( E = \text{“sandwich preparation”}, s_1 = \text{“chop bread”}, s_2 = \text{“choose condiment”}, \ldots \)
The Multiplication Rule

• Suppose that $E$ is some experiment that is conducted through $k$ sequential steps $s_1, s_2, \ldots, s_k$, where every $s_i$ can be conducted in $n_i$ different ways.
  • Example: $E$ = “sandwich preparation”, $s_1$ = “chop bread”, $s_2$ = “choose condiment”, ...

• Then, the total number of ways that $E$ can be conducted in is

$$\prod_{i=1}^{k} n_i = n_1 \times n_2 \times \cdots \times n_k$$
A Familiar Example

• How many subsets are there of a set of 4 elements?
• Example: \{a, b, c, d\}
  • a: in or out. 2 choices.
  • b: in or out. 2 choices.
  • c: in or out. 2 choices.
  • d: in or out. 2 choices.
A Familiar Example

• How many subsets are there of a set of 4 elements?
• Example: \{a, b, c, d\}
  • a: in or out. 2 choices.
  • b: in or out. 2 choices.
  • c: in or out. 2 choices.
  • d: in or out. 2 choices.
\[2 \times 2 \times 2 \times 2 = 2^4 = 16\text{ subsets.}\]
A Familiar Example

• How many subsets are there of a set of 4 elements?
• Example: \{a, b, c, d\}
  • a: in or out. 2 choices.
  • b: in or out. 2 choices.
  • c: in or out. 2 choices.
  • d: in or out. 2 choices.
  \[2 \times 2 \times 2 \times 2 = 2^4 = 16\] subsets.

• Generalization: there are \(2^n\) subsets of a set of size \(n\).
  • But you already knew this.
Permutations

• Consider the string “machinery”.

• A permutation of “machinery” is a string which results by re-organizing the characters of “machinery” around.
  • Examples: chyirenma, hcyranemi, machinery (!)
  • Question: How many permutations of “machinery” are there?
# Permutations

machinery

9 options for ‘m’
# Permutations

`machinery`

9 options for ‘m’
# Permutations

machinery

8 options for 'a'
# Permutations

machinery

8 options for ‘a’
# Permutations

machinery

7 options for ‘c’...

___m___a___
# Permutations

machinery

_ _ m _ _ c a _ _ 7 options for ‘c’...
# Permutations

machinery

_ _ m _ _ c a _ _

6 options for ‘h’...
# Permutations

machinery

6 options for ‘h’...

h_ m_ c a_
# Permutations

machinery

5 options for ‘i’

h _ m _ _ c a _ _
# Permutations

machinery

5 options for ‘i’
# Permutations

**machinery**

4 options for ‘n’

- **h m c a i**
# Permutations

machinery

h _ m _ n c a _ i

4 options for ‘n’
# Permutations

machinery

h_m_nca_i

3 options for ‘e’
# Permutations

machinery

3 options for ‘e’
# Permutations

machinery

2 options for ‘r’
# Permutations

machinery

2 options for ‘r’

hemn carj
# Permutations

machinery

hemnca

1 option for ‘y’
# Permutations

machinery

1 option for ‘y’
# Permutations

```
machinery
```

1 option for ‘y’

```
heymncari
```

Total #possible permutations = 9 × 8 × ⋯ × 2 × 1 = 9! = 362880
# Permutations

Total #possible permutations = \(9 \times 8 \times \cdots \times 2 \times 1 = 9! = 362880\)

That’s a lot! (Original string has length 9)
# Permutations

machinery

1 option for ‘y’

he my nc ar i

Total #possible permutations = $9 \times 8 \times \cdots \times 2 \times 1 = 9! = 362880$

In general, for a string of length $n$ we have ourselves $n!$ different permutations!

That’s a lot! (Original string has length 9)
Permutations

• Now, consider the string “puzzle”.
• How many permutations are there of this string?
• Note that two letters in puzzle are the same.
Permutations

• Now, consider the string “puzzle”.
• How many permutations are there of this string?
• Note that two letters in puzzle are the same.
  • Call the first \( z \) \( z_1 \) and the second \( z \) \( z_2 \)
• So, one permutation of \( puz_1z_2le \) is \( puz_2z_1le \)
  • But this is clearly equivalent to \( puz_1z_2le \), so we wouldn’t want to count it!
  • So clearly the answer is not \( 6! \) (6 is the length of “puzzle”)
  • What is the answer?
Thought Experiment

• Pretend the two ‘z’s in “puzzle” are different, e.g \( z_1, z_2 \)
  • Then, 6! permutations, as discussed
  • Now we have the “equivalent” permutations, for instance

\[
\begin{align*}
  z_1 & \text{pul} z_2 e \\
  z_2 & \text{pul} z_1 e
\end{align*}
\]

• We want to not doublecount these!
Thought Experiment

\[ z_1pulz_2e \]
\[ z_2pulz_1e \]

We want to **not doublecount** such permutations!

- Then, we need to stop pretending that the ‘z’s are *different*
  - **Bad news:** 6! is overcount 😞
  - **Good news:** 6! is an overcount in a precise way! 😊 Everything is counted exactly twice!
Thought Experiment

\[ z_1pulz_2e \]
\[ z_2pulz_1e \]

We want to **not doublecount** such permutations!

- Then, we need to stop pretending that the ‘z’\’s are **different**
  - **Bad news:** 6! is overcount 😞
  - **Good news:** 6! is an overcount in a precise way! 😊 Everything is counted **exactly twice**!
- **Answer:** \[\frac{6!}{2}\]
Permutations

• Now, consider the string “scissor”.
• How many permutations of “scissor” are there?
• Note that three letters in “scissor” are the same.
  • As previously discussed, the answer cannot be 7! (7 is the length of “scissor”)
Permutations

• Now, consider the string “scissor”.
• How many permutations of “scissor” are there?
• Note that three letters in “scissor” are the same.
  • As previously discussed, the answer cannot be 7! (7 is the length of “scissor”)
  • Observe all the possible positions of the various ‘s’s:
    • $s_1cis_2s_3or$
    • $s_1cis_3s_2or$
    • $s_2cis_1s_3or$
    • $s_2cis_3s_1or$
    • $s_3cis_1s_2or$
    • $s_3cis_2s_1or$
Permutations

• Now, consider the string “scissor”.
• How many permutations of “scissor” are there?
• Note that three letters in “scissor” are the same.
  • As previously discussed, the answer cannot be $7!$ ($7$ is the length of “scissor”)
  • Observe all the possible positions of the various ‘$s$’s:
  
  $s_1cis_2s_3or$
  $s_1cis_3s_2or$
  $s_2cis_1s_3or$
  $s_2cis_3s_1or$
  $s_3cis_1s_2or$
  $s_3cis_2s_1or$

  $3! = 6$ different ways to arrange those $3$ ‘$s$’s
Final Answer

• Think of it like this: *How many times can I fit essentially the same string into the number of permutations of the original string?*

• Therefore, the total #permutations when not assume different ‘s’s is

\[
\frac{7!}{3!} = \frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}{1 \times 2 \times 3} = 20 \times 42 = 840
\]
Complex Overcounting

• Consider now the string “onomatopoeia”.
• 12 letters, with 4 ‘o’s, 2 ‘a’s
• Considering the characters being different, we have:

\[ o_1 n o_2 m a t o_3 p o_4 e i a, \]
Complex Overcounting

• Consider now the string “onomatopoeia”.
• 12 letters, with 4 ‘o’s, 2 ‘a’s
• Considering the characters being different, we have:

\[ o_1n_o_2ma_t_0_3p_0_4eia, \]
\[ o_1n_o_2ma_t_0_4p_0_3eia, \]
Complex Overcounting

• Consider now the string “onomatopoeia”.
• 12 letters, with 4 ‘o’s, 2 ‘a’s
• Considering the characters being different, we have:

\[ o_1 n o_2 m a t o_3 p o_4 e i a, \]
\[ o_1 n o_2 m a t o_4 p o_3 e i a, \]
\[ o_1 n o_3 m a t o_4 p o_2 e i a, \]
Complex Overcounting

• Consider now the string “onomatopoeia”.
• 12 letters, with 4 ‘o’s, 2 ‘a’s
• Considering the characters being different, we have:

How many such positionings of the ‘o’s are possible?

6 12 16

Something Else
Complex Overcounting

• Consider now the string “onomatopoeia”.
• 12 letters, with 4 ‘o’s, 2 ‘a’s
• Considering the characters being different, we have:

How many such positionings of the ‘o’s are possible?

\[ 6, 12, 16 \]

\[ o_1 n o_2 m a t o_3 p o_4 e i a, \]
\[ o_1 n o_2 m a t o_4 p o_3 e i a, \]
\[ o_1 n o_3 m a t o_4 p o_2 e i a, \]
\[ \ldots \]

\[ 4! = 24 \text{ different ways.} \]
Complex Overcounting

• However, we also have the two ‘a’s to consider!
• Fortunately, those equivalent permutations are simpler to count:

\[ onoma_1 topoeia_2 \]
\[ onoma_2 topoeia_1 \]
Complex Overcounting

• However, we also have the two ‘a’s to consider!
• Fortunately, those equivalent permutations are simpler to count:

\[ onoma_1 \text{topoei}a_2 \]
\[ onoma_2 \text{topoei}a_1 \]

• Key: for every one of these two (equivalent) permutations, we have 4! equivalent permutations because of the ‘o’s! (MULTIPLICATION RULE)
Complex Overcounting

• However, we also have the two ‘a’s to consider!
• Fortunately, those equivalent permutations are simpler to count:

\[
onoma_1 \topoeia_2 \\
onoma_2 \topoeia_1
\]

• Key: **for every one** of these two (equivalent) permutations, we have 4! equivalent permutations because of the ‘o’! *(MULTIPLICATION RULE)*

• Final answer:

\[
\#\text{permutations} = \frac{12!}{4! \cdot 2!} = \frac{5 \cdot 6 \cdot ... \cdot 11 \cdot 12}{2} = 5 \cdot 6^2 \cdot ... \cdot 10 \cdot 11 = 9,979,200
\]
Important “Pedagogical” Note

• In the previous problem, we came up with the quantity

\[
\frac{12!}{4! \cdot 2!} = 9,979,200
\]
Important “Pedagogical” Note

• In the previous problem, we came up with the quantity

\[ \frac{12!}{4! \cdot 2!} = 9,979,200 \]

• How you should answer in an exam: \[ \frac{12!}{4! \cdot 2!} \]

• Don’t perform computations, like 9,979,200
  • Helps you save time and us when grading 😊
For You!

• Consider the word “bookkeeper” (according to this website, the only unhyphenated word in English with three consecutive repeated letters)

• How many non-equivalent permutations of “bookkeeper” exist?
For You!

• Consider the word “bookkeeper” (according to this website, the only unhyphenated word in English with three consecutive repeated letters)

• How many non-equivalent permutations of “bookkeeper” exist?

$$\frac{10!}{2! \cdot 2! \cdot 3!}$$

Don’t forget the third ‘e’!
More Practice

• What about the #non-equivalent permutations for the word combinatorics
More Practice

• What about the non-equivalent permutations for the word

\[ \frac{13!}{2! \cdot 2! \cdot 2!} = \ldots \]
General Template

• Total # permutations of a string $\sigma$ of letters of length $n$ where there are $n_a$ 'a's, $n_b$ 'b's, $n_c$ 'c's, ... $n_z$ 'z's

$$\frac{n!}{n_a! \times n_b! \times \cdots \times n_z!}$$
General Template

• Total # permutations of a string $\sigma$ of letters of length $n$ where there are $n_a$ 'a's, $n_b$ 'b's, $n_c$ 'c's, ... $n_z$ 'z's

$$\frac{n!}{n_a! \times n_b! \times \cdots \times n_z!}$$

• Claim: This formula is problematic when some letter (a, b, ..., z) is **not** contained in $\sigma$

Yes  No
General Template

• Total # permutations of a string \( \sigma \) of letters of length \( n \) where there are \( n_a \) 'a's, \( n_b \) 'b's, \( n_c \) 'c's, ... \( n_z \) 'z's

\[
\frac{n!}{n_a! \times n_b! \times \cdots \times n_z!}
\]

• Claim: This formula is problematic when some letter (a, b, ..., z) is **not** contained in \( \sigma \)

Remember: 
0! = 1
\(r\)-permutations

- Warning: permutations (as we’ve talked about them) are best presented with strings.
- \(r\)-permutations: Those are best presented with sets.
  - Note that \(r \in \mathbb{N}\)
  - So we can have 2-permutations, 3-permutations, etc
$r$-permutations: Example

• I have ten people.

• My goal: pick three people for a picture, where order of the people matters.
\( r \)-permutations: Example

• I have ten people.

• My goal: pick three people for a picture, where order of the people matters.

• Examples: shortest-to-tallest or tallest-to-shortest or something-in-between
$r$-permutations: Example

- I have ten people.

- My goal: pick three people for a picture, where order of the people matters.
- Examples: Jenny-Fred-Bob or Fred-Jenny-Bob or Fred-Bob-Jenny
$r$-permutations: Example

• I have ten people.

• My goal: pick three people for a picture, where order of the people matters.

• In how many ways can I pick these people?
$r$-permutations: Example

I need three people for this photo. You guys figure out your order.
I need three people for this photo. You guys figure out your order.
I need three people for this photo. You guys figure out your order.

\( r \)-permutations: Example

10 ways to pick the first person...
$r$-permutations: Example

I need three people for this photo. You guys figure out your order.

10 ways to pick the first person...
I need three people for this photo. You guys figure out your order.

10 ways to pick the first person...
I need three people for this photo. You guys figure out your order.

9 ways to pick the second person...
I need three people for this photo. You guys figure out your order.

$r$-permutations: Example

9 ways to pick the second person...
I need three people for this photo. You guys figure out your order.

9 ways to pick the second person...
I need three people for this photo. You guys figure out your order.

8 ways to pick the third person...
$r$-permutations: Example

I need three people for this photo. You guys figure out your order.

8 ways to pick the third person...
$r$-permutations: Example

I need three people for this photo. You guys figure out your order.

8 ways to pick the third person...
I need three people for this photo. You guys figure out your order.

For a total of $10 \times 9 \times 8 = 720$ ways.
I need three people for this photo. You guys figure out your order.

For a total of \(10 \times 9 \times 8 = 720\) ways.

Note: \(10 \times 9 \times 8 = \frac{10!}{(10-3)!}\)
Example on Books

• Clyde has the following books on his bookshelf
  • Epp, Rosen, Hughes, Bogart, Davis, Shaffer, Sellers, Scott

• Jason wants to borrow any 5 of them and read them in the order he picks them in.

• In how many ways can Jason get smart by reading those books?
Example on Books

• Clyde has the following books on his bookshelf
  • Epp, Rosen, Hughes, Bogart, Davis, Shaffer, Sellers, Scott

• Jason wants to borrow any 5 of them and read them in the order he picks them in.

• In how many ways can Jason get smart by reading those books?

$$\frac{8!}{(8 - 5)!} = \frac{8!}{3!}$$
• Let $n, r \in \mathbb{N}$ such that $0 \leq r \leq n$. The total ways in which we can select $r$ elements from a set of $n$ elements \textit{where order matters} is equal to:

$$P(n, r) = \frac{n!}{(n - r)!}$$
General Formula

• Let $n, r \in \mathbb{N}$ such that $0 \leq r \leq n$. The total ways in which we can select $r$ elements from a set of $n$ elements where order matters is equal to:

$$P(n, r) = \frac{n!}{(n - r)!}$$

“$P$” for permutation. This quantity is known as the $r$-permutations of a set with $n$ elements.
Pop Quizzes

1) $P(n, 1) = \cdots \quad 0 \quad 1 \quad n \quad n!$
Pop Quizzes

1) \( P(n, 1) = \ldots \)

• Two ways to convince yourselves:
  • Formula: \( \frac{n!}{(n-1)!} = n \)
  • Semantics of \( r \)-permutations: In how many ways can I pick 1 element from a set of \( n \) elements? Clearly, I can pick any one of \( n \) elements, so \( n \) ways.
Pop Quizzes

2) \( P(n, n) = \cdots \quad 0 \quad 1 \quad n \quad n! \)
Pop Quizzes

2) \( P(n, n) = \cdots \)  

\[
P(n, n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!
\]

• Again, two ways to convince ourselves:
  • Formula: \( \frac{n!}{(n-n)!} = \frac{n!}{0!} \)
  • Semantics: \( n! \) ways to pick all of the elements of a set and put them in order!
Pop Quizzes

3) $P(n, 0) = \cdots \ 0 \ 1 \ n \ n!$
3) \( P(n, 0) = \cdots \)

\[
P(n, 0) = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1
\]

Again, two ways to convince ourselves:

- **Formula:** 
  \[
  \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1
  \]

- **Semantics:** Only one way to pick nothing: just pick nothing and leave!
Practice

1. How many MD license plates are possible to create?
1. How many MD license plates are possible to create? $26^2 \cdot 10^5$
Practice

1. How many MD license plates are possible to create? $26^2 \cdot 10^5$
2. How many ATM PINs are possible?
Practice

1. How many MD license plates are possible to create? $26^2 \cdot 10^5$
2. How many ATM PINs are possible? $10^4$
Practice

1. How many MD license plates are possible to create? $26^2 \cdot 10^5$
2. How many ATM PINs are possible? $10^4$
3. How many words of length 10 can I construct from the English alphabet, where letters can be chosen:
Practice

1. How many MD license plates are possible to create? \(26^2 \cdot 10^5\)
2. How many ATM PINs are possible? \(10^4\)
3. How many words of length 10 can I construct from the English alphabet, where letters can be chosen:
   a) With replacement (as in, I can reuse letters)
Practice

1. How many MD license plates are possible to create? $26^2 \cdot 10^5$

2. How many ATM PINs are possible? $10^4$

3. How many words of length 10 can I construct from the English alphabet, where letters can be chosen:
   a) With replacement (as in, I can reuse letters) $26^{10}$
Practice

1. How many MD license plates are possible to create? $26^2 \cdot 10^5$

2. How many ATM PINs are possible? $10^4$

3. How many words of length 10 can I construct from the English alphabet, where letters can be chosen:
   a) With replacement (as in, I can reuse letters) $26^{10}$
   b) Without replacement (as in, I cannot reuse letters)
1. How many MD license plates are possible to create? \(26^2 \cdot 10^5\)
2. How many ATM PINs are possible? \(10^4\)
3. How many words of length 10 can I construct from the English alphabet, where letters can be chosen:
   a) With replacement (as in, I can reuse letters) \(26^{10}\)
   b) Without replacement (as in, I cannot reuse letters) \(P(26, 10) = \frac{26!}{16!}\)
Practice

1. How many MD license plates are possible to create? $26^2 \cdot 10^5$

2. How many ATM PINs are possible? $10^4$

3. How many words of length 10 can I construct from the English alphabet, where letters can be chosen:
   a) With replacement (as in, I can reuse letters) $26^{10}$
   b) Without replacement (as in, I cannot reuse letters) $P(26, 10) = \frac{26!}{16!}$

Remember these phrases!
Combinations (that “n choose r” stuff)

• Earlier, we discussed this example:

I need three people for this photo. You guys figure out your order.

• Our goal was to pick three people for a picture, where order of the people mattered.
Earlier, we discussed this example:

We now change this setup to forming a PhD defense committee (also 3 people).

In this setup, does order matter?
Combinations (that “n choose r” stuff)

• Earlier, we discussed this example:

• We now change this setup to forming a PhD defense committee (also 3 people).
• In this setup, does order matter?
Combinations (that “n choose r” stuff)
Combinations (that “n choose r” stuff)

We can make this selection in $P(10, 3)$ ways...
Combinations (that “n choose r” stuff)

We can make this selection in $P(10, 3)$ ways... but since order doesn’t matter, we have $3!$ permutations of these people that are equivalent.
Combinations (that “n choose r” stuff)

We can make this selection in $P(10, 3)$ ways... but since order doesn’t matter, we have 3! permutations of these people that are equivalent.
Combinations (that “n choose r” stuff)

We can make this selection in $P(10, 3)$ ways... but since order doesn’t matter, we have $3!$ permutations of these people that are equivalent.

Overcount 😞
Combinations (that “n choose r” stuff)

We can make this selection in $P(10, 3)$ ways... but since order doesn’t matter, we have $3!$ permutations of these people that are equivalent.
Combinations (that “n choose r” stuff)

Overcount 😞
In a precise way 😊

We can make this selection in $P(10, 3)$ ways... but since order doesn’t matter, we have $3!$ permutations of these people that are equivalent.
Closer Analysis of Example

• Note that essentially we are asking you: Out of a set of 10 people, how many subsets of 3 people can I retrieve?
(\(\binom{n}{r}\)) Notation

• The quantity

\[
\frac{P(10, 3)}{3!}
\]

is the number of \textit{3-combinations} from a set of size 10, denoted thus:

\[
\binom{n}{3}
\]

and pronounced “n choose 3”.

\( \binom{n}{r} \) Notation

- Let \( n, r \in \mathbb{N} \) with \( 0 \leq r \leq n \)
- Given a set \( A \) of size \( n \), the total number of subsets of \( A \) of size \( r \) is:

\[
\binom{n}{r} = \frac{n!}{r! (n-r)!}
\]
(n \choose r) \text{ Notation}

• Let \( n, r \in \mathbb{N} \) with \( 0 \leq r \leq n \)

• Given a set \( A \) of size \( n \), the total number of subsets of \( A \) of size \( r \) is:

\[
{n \choose r} = \frac{n!}{r! (n-r)!}
\]

• Pop quiz: \((\forall n, r \in \mathbb{N})[(0 \leq r \leq n) \Rightarrow (\binom{n}{r} \leq P(n, r))]\)

True  False
\( \binom{n}{r} \) Notation

• Let \( n, r \in \mathbb{N} \) with \( 0 \leq r \leq n \)

• Given a set \( A \) of size \( n \), the total number of subsets of \( A \) of size \( r \) is:

\[
\binom{n}{r} = \frac{n!}{r! \ (n-r)!}
\]

• Pop quiz: \( \forall n, r \in \mathbb{N} \) \( (0 \leq r \leq n) \Rightarrow \left( \binom{n}{r} \leq P(n, r) \right) \]

Recall that

\[
\binom{n}{r} = \frac{p(n,r)}{r!} \quad \text{and} \quad r! \geq 1
\]
Quiz
1. \( \binom{n}{1} = \)
1. \( \binom{n}{1} = n \)
1. \( \binom{n}{1} = n \)

2. \( \binom{n}{n} = \)
1. \( \binom{n}{1} = n \)

2. \( \binom{n}{n} = 1 \) (Note how this differs from \( P(n, n) = n! \))
1. \( \binom{n}{1} = n \)

2. \( \binom{n}{n} = 1 \) (Note how this differs from \( P(n, n) = n! \))

3. \( \binom{n}{0} = \)
1. $\binom{n}{1} = n$
2. $\binom{n}{n} = 1$ (Note how this differs from $P(n, n) = n!$)
3. $\binom{n}{0} = 1$
STOP
RECORDING