Midterm Review

250H

k-nomial

$$(x_1 + x_2 + \cdots + x_k)^n = \sum_{\substack{a_1+a_2+\cdots+a_k=n \\ 0\le a_1,a_2,\ldots,a_k\le n}} \frac{n!}{a_1!\,a_2!\ldots a_k!} x_1^{a_1} x_2^{a_2} \ldots x_k^{a_k}$$

$$\Leftrightarrow \left(\sum_{i=1}^{k} x_i\right)^n = \sum_{\substack{a_1+a_2+\cdots+a_k=n \\ 0\le a_1,a_2,\ldots,a_k\le n}} \frac{n!}{\prod_{i=1}^{k} a_i!} \prod_{i=1}^{k} x_i^{a_i}$$

Card Combinatorial Problems

You have a standard deck of 52 cards. Each card is a unique pair of suit and rank. There are 4 suits – heart, diamond, club, and spade. There are 13 ranks – Ace, 2, 3, . . . , 9, Jack, Queen, King. You pick 4 cards from the deck, and place them in the order picked.

(a) How many 4-card drawings (ordered) are possible?

(b) What if the second card must be the Ace of Spades?

(c) What if the second card must **not** be the Ace of Spades?

(d) What if the first and second cards must be the same suit?

(e) What if all cards must be the same suit?

(f) What if all cards must be the same rank?

$\binom{52}{4}$
$\binom{51}{1}\binom{50}{2}$
$\binom{52}{4} - \binom{51}{1}\binom{50}{2}$
$(52)(12)\binom{50}{2}$
$4 * \binom{13}{4}$
$13 * 4!$

Card Combinatorial Problems

Unlike the previous game, in this game, you draw five cards to form a *hand*. The hand is *unordered*. For instance, these two count as the *same* hand, despite being in different orders:

$$\mathbf{4}\diamondsuit,\ \mathbf{7}\clubsuit,\ \mathbf{J}\diamondsuit,\ \mathbf{3}\heartsuit,\ \mathbf{A}\spadesuit$$

$$\mathbf{J}\diamondsuit,\ \mathbf{7}\clubsuit,\ \mathbf{A}\spadesuit,\ \mathbf{4}\diamondsuit,\ \mathbf{3}\heartsuit$$

You are curious about how many hands you are able to create under certain conditions.

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You are curious about how many hands you are able to create under certain conditions.

(b) How many possible hands are there containing the card $\mathbf{A}\spadesuit$ (the Ace of Spades)?

$\binom{51}{4}$

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$$\mathbf{J}\diamondsuit,\ \mathbf{7}\clubsuit,\ \mathbf{A}\spadesuit,\ \mathbf{4}\diamondsuit,\ \mathbf{3}\heartsuit$$

You are curious about how many hands you are able to create under certain conditions.

(c) How many possible hands are there that contain at least one **J** (Jack) or at least *two* **K** (Kings)?

$$\underline{4 \cdot \binom{51}{4} + \binom{4}{2} \cdot \binom{50}{3} - 4 \cdot \binom{4}{2} \cdot \binom{49}{2}}$$

Card Combinatorial Problems

A *full house* is a hand that consists of three cards that share one rank, and two cards that share a different rank. The suits do not matter, but changing the suits creates a different, unique full house. This is an example of a valid full house:

$$\mathbf{7}\diamondsuit,\ \mathbf{2}\spadesuit,\ \mathbf{7}\spadesuit,\ \mathbf{7}\clubsuit,\ \mathbf{2}\diamondsuit$$

Card Combinatorial Problems

A *full house* is a hand that consists of three cards that share one rank, and two cards that share a different rank. The suits do not matter, but changing the suits creates a different, unique full house. This is an example of a valid full house:

$$\mathbf{7}\diamondsuit,\ \mathbf{2}\spadesuit,\ \mathbf{7}\spadesuit,\ \mathbf{7}\clubsuit,\ \mathbf{2}\diamondsuit$$

(d) How many possible full houses are there?

$$\underline{\binom{13}{2} \cdot 2! \cdot \binom{4}{3} \cdot \binom{4}{2}}$$

Card Combinatorial Problems

A *full house* is a hand that consists of three cards that share one rank, and two cards that share a different rank. The suits do not matter, but changing the suits creates a different, unique full house. This is an example of a valid full house:

(e) How many possible full houses are there where both ranks that are present *sum* to exactly 8? (**A** cards count as the number one. The cards **J**, **Q**, and **K** count as the number 10, so you can ignore them.)

$$\underline{3 \cdot 2! \cdot \binom{4}{3} \cdot \binom{4}{2}}$$

Remark: There are only 3 possible distinct-rank combinations summing to 8: (**A**, **7**), (**2**, **6**), and (**35**).

Bayes Theorem

$$\Pr(A|B) \; = \; \Pr(B|A) * \frac{\Pr(A)}{\Pr(B)}$$

$$\Pr(A|B) \; = \; \Pr(B|A) * \frac{\Pr(A)}{\Pr(B|\mathrm{A})\Pr(A) + \; \Pr(B| \sim A)\Pr(\sim A)}$$

1. You like going outside, for whatever reason. Yet, you hate going outside when it's raining. You know that if it rained during the day then it was cloudy that same morning with probability $\frac{1}{2}$. You know that it's cloudy in the morning with probability $\frac{1}{3}$. You know that it's rainy during the day with probability $\frac{1}{5}$. Calculate the probability of it raining today if it is cloudy this morning. Should you go outside?

By Bayes',

$$\begin{aligned} Pr(R \mid C) &= \frac{Pr(R)Pr(C \mid R)}{Pr(C)} \\ &= \frac{\frac{1}{5} \cdot \frac{1}{2}}{\frac{1}{3}} = \frac{3}{10} = 30\% \end{aligned}$$

With 30% chance of it raining today, you *probably* could risk it. Should you, though? Nah – going outside is scary enough.

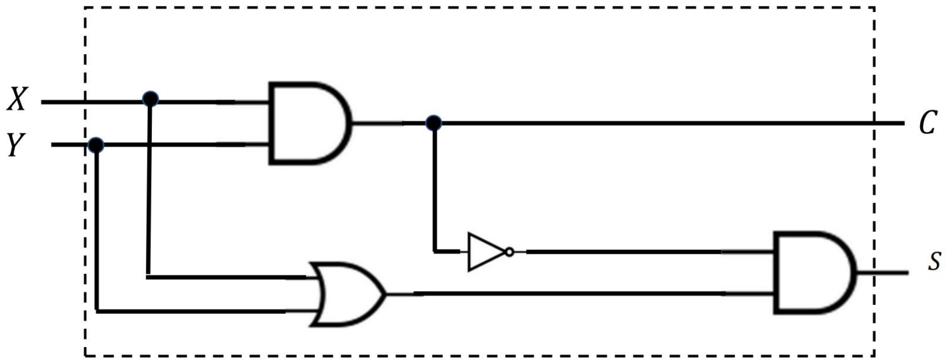
2. *Rehash the lecture example* – The probability of being covid positive is 0.001, the probability of a covid positive patient having a positive test is 0.99 (the true-positive rate), and the probability of a covid negative patient having a positive test is 0.05 (the false-positive rate). Calculate the probability of being covid positive *given* you tested positive.

By Bayes',

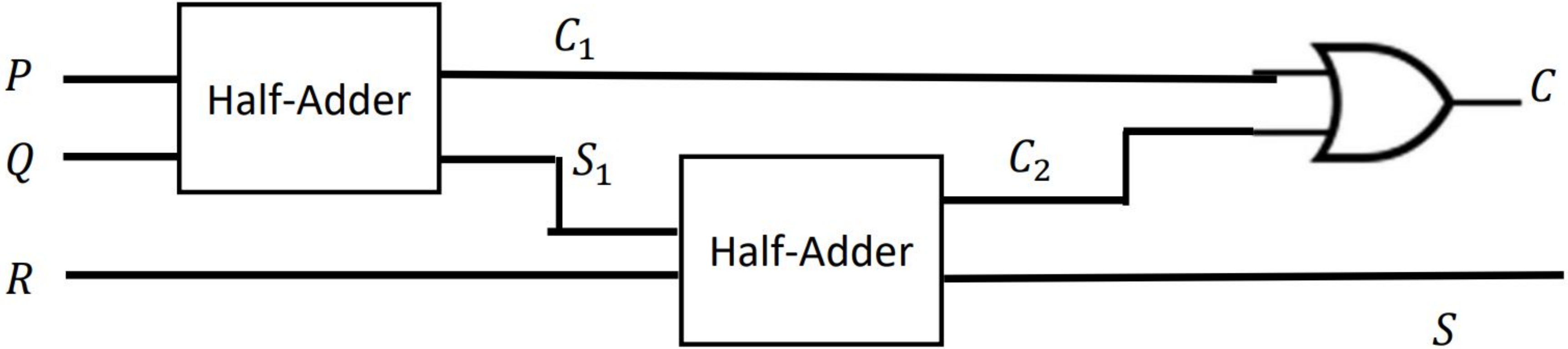
$$\begin{aligned}
Pr(C+ \mid t+) &= \frac{Pr(C+)Pr(t+ \mid C+)}{Pr(t+)} \\
&= \frac{Pr(C+)Pr(t+ \mid C+)}{Pr(t+ \mid C+)Pr(C+) + Pr(t+ \mid C-)Pr(C-)} \\
&= \frac{0.001 \cdot 0.99}{0.99 \cdot 0.001 + 0.05 \cdot (1 - 0.001)} \\
&\approx 0.0194
\end{aligned}$$

Half Adders

X	Y	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0



Full Adders



Hat Check

n people give their hats to a hat check person.

The hat check person gives people their hats RANDOMLY.

What is Prob NOBODY gets their correct hat?

$$\text{P(some)} =$$

$$\frac{n!}{1!(n-1)!}\frac{(n-1)!}{n!} - \frac{n!}{2!(n-2)!}\frac{(n-2)!}{n!} \pm \frac{1}{n!}$$

$$= 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} \pm \frac{1}{n!}$$

Tips for Taking the Exam

1. Make a cheat sheet
2. Do not wait to ask questions on problems that you do not understand
3. Do not rely on patterns from the HW
4. Do not look at material 1 hour before the exam
5. Make sure you eat before the exam or have a snack while taking it
6. Have water next to you
7. Have an emotional support rubber duck
8. Breathe!!!!