Some More Combinatorics

250H
How many solutions are there to $x_1 + \ldots + x_k = n$, where $x_1, \ldots, x_k$ in $\{0,1,2,\ldots\}$?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$
Balls and Lines \ Stars and Bars

How many ways there are to put \( n \) indistinguishable balls into \( k \) distinguishable bins?

\[
* | *** | **
\]
Stars and Bars

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We can view this as permutations of \( k-1 \) lines and \( n \) balls.
Stars and Bars

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\begin{array}{c|c|c|c|c}
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\end{array}
\]

We can view this as permutations of \( k-1 \) lines and \( n \) balls.

\[
\frac{(n+k-1)!}{n!(k-1)!} = \binom{n+k-1}{k-1} = \binom{n+k-1}{n}
\]
How many solutions are there to $x_1 + \ldots + x_k = n$, where $x_1, \ldots, x_k$ in \{0,1,2,\ldots\}?

$$x_1 + x_2 + x_3 + x_4 + x_5 = 40$$

$$\frac{(n+k-1)!}{n!(k-1)!} = \frac{(40+5-1)!}{40!(5-1)!} = 135751$$
Stars and bars: What if all the $x_i$ are $\geq 1$?
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Then we have $k-1$ bars and $n-k$ stars.
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Then we have $k-1$ bars and $n-k$ stars.

$$\frac{((n-k)+(k-1)-1)!}{(n-k)!((k-1)-1)!} = \frac{n-2!}{(n-k)!(k-2)!}$$