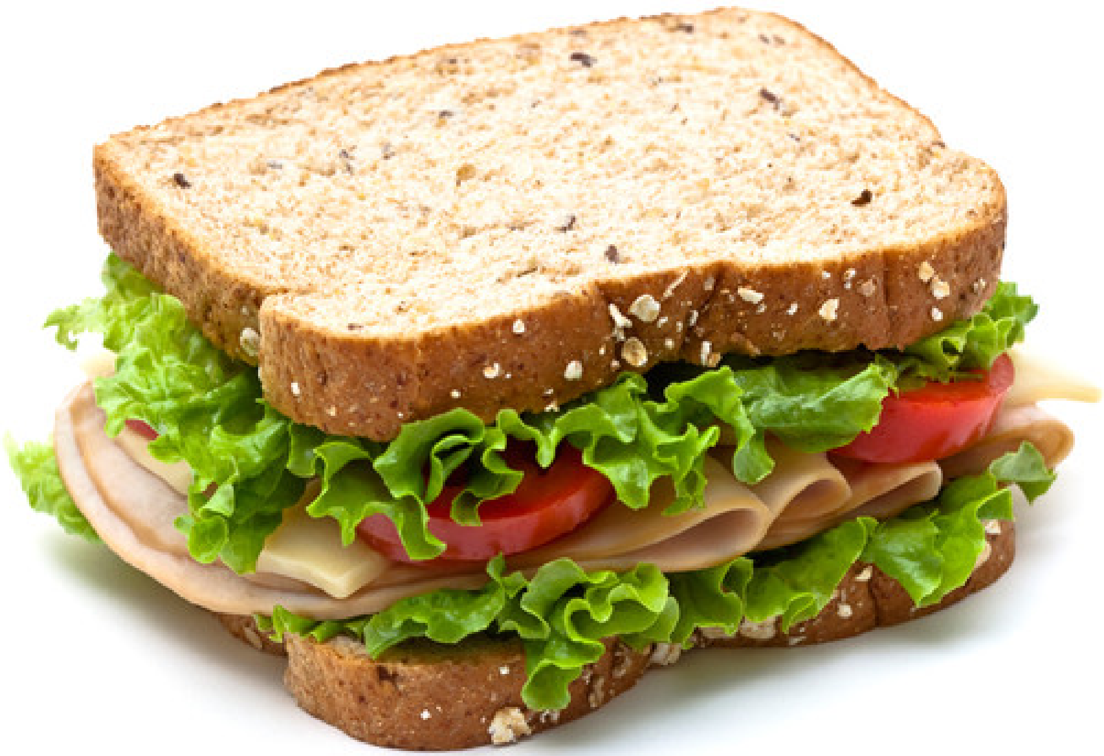
START RECORDING

Intro to Combinatorics
(“that n choose 2 stuff”)

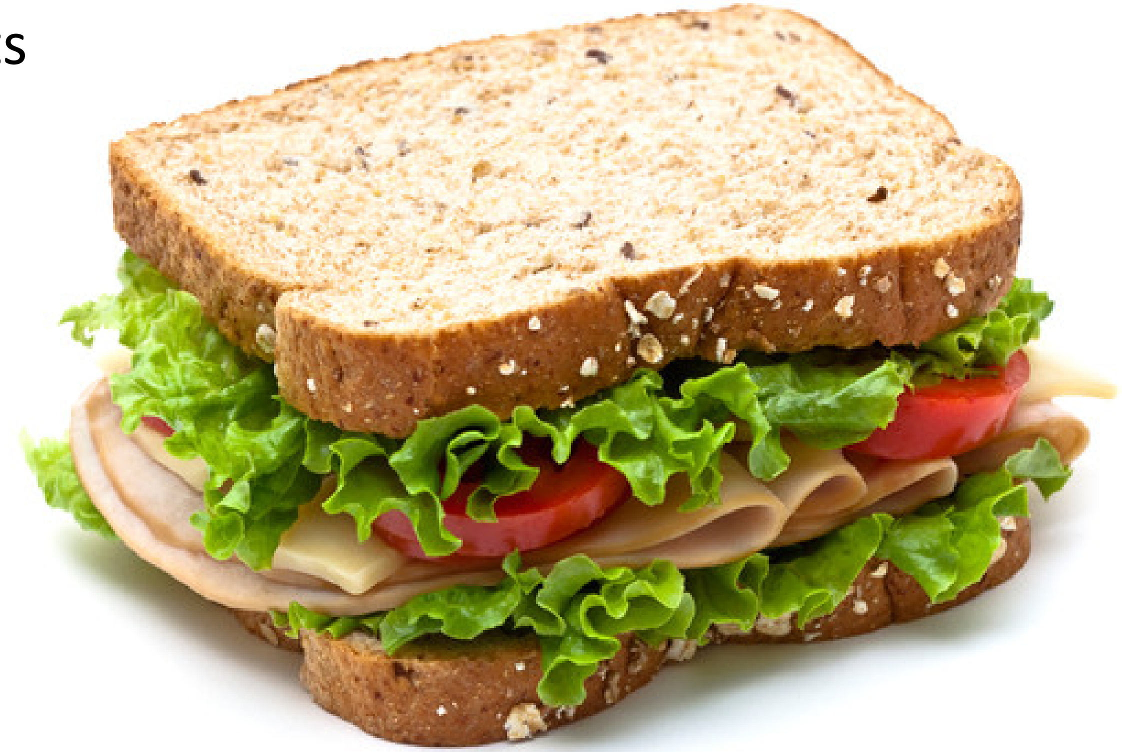
CMSC 250

Jason’s sandwich



Jason’s Sandwich

- Suppose that Jason has the following ingredients to make a sandwich with:
  - White or black bread
  - Butter, Mayo or Honey Mustard
  - Romaine Lettuce, Spinach, Kale
  - Bologna, Ham or Turkey
  - Tomato or egg slices



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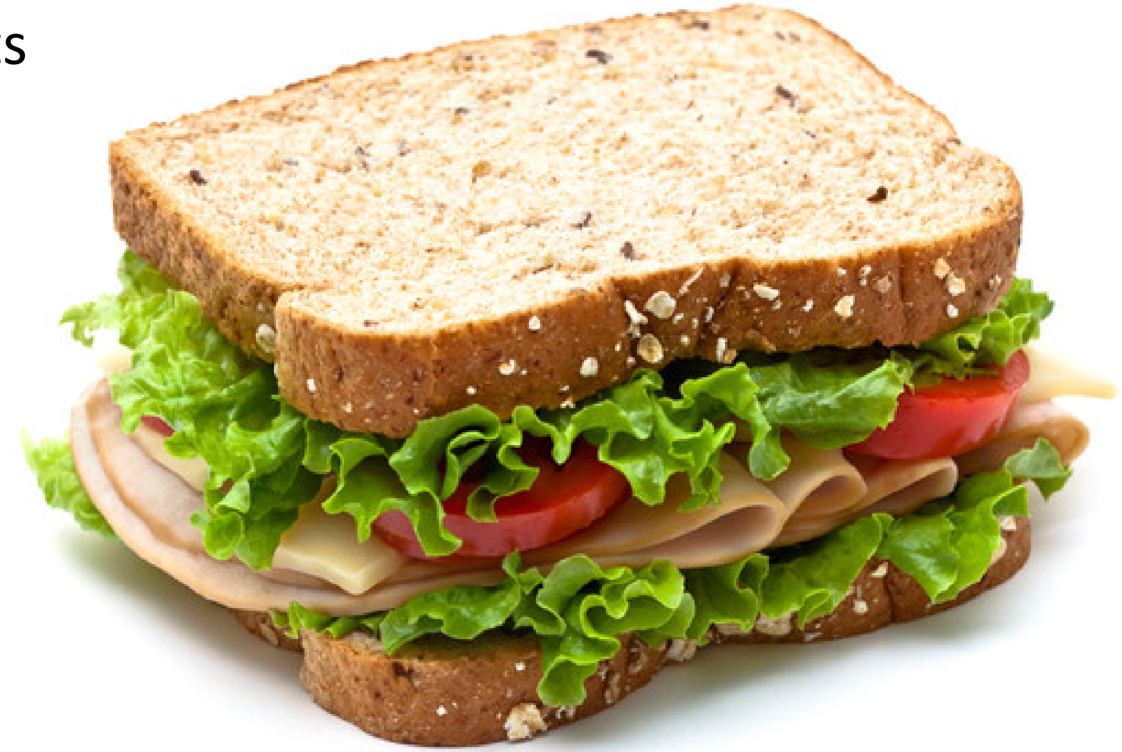
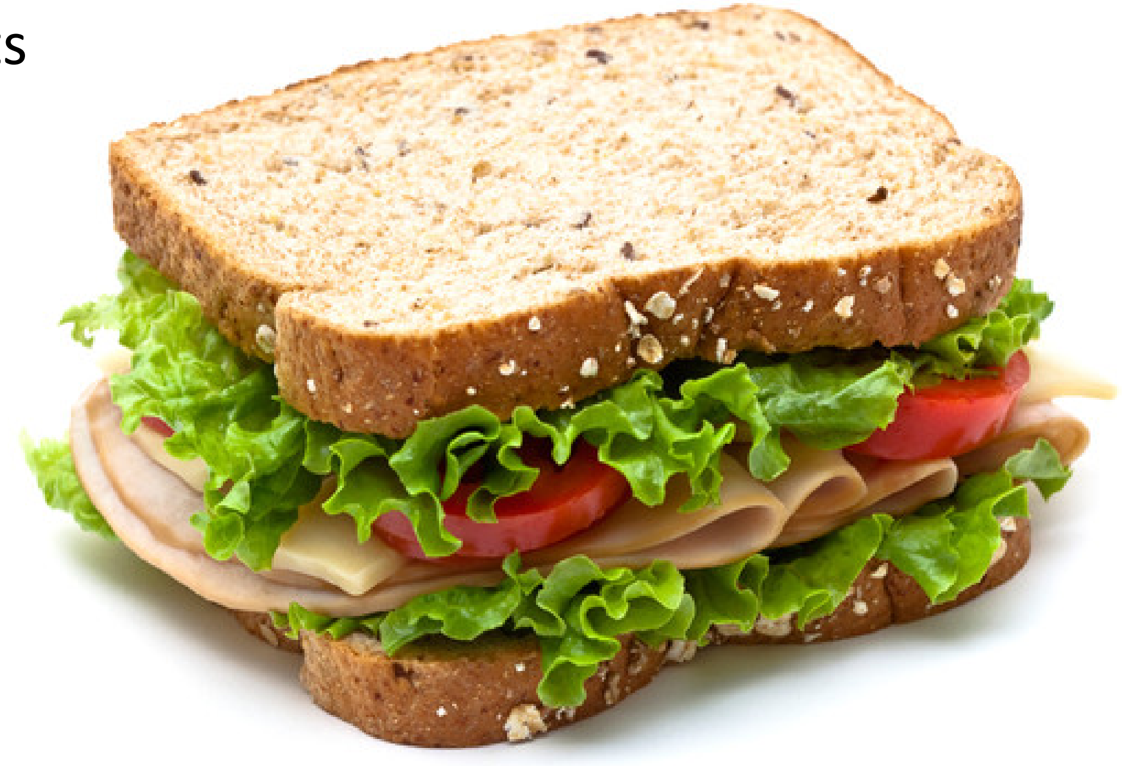
- **How many different sandwiches can Jason make?**



Jason’s Sandwich

- Suppose that Jason has the following ingredients to make a sandwich with:
  - White or black bread 2 options
  - Butter, Mayo or Honey Mustard 3 options
  - Romaine Lettuce, Spinach, Kale 3 options
  - Bologna, Ham or Turkey 3 options
  - Tomato or egg slices 2 options
- **How many different sandwiches can Jason make?**
  - $2 \times 3 \times 3 \times 3 \times 2 = 4 \times 27 = 108$



The Multiplication Rule

- Suppose that E is some experiment that is conducted through k sequential steps $s_1, s_2, \ldots, s_k$, where every s_i can be conducted in n_i different ways.

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  - Example: $E = \text{“}sandwich\ preparation\text{”}, s_1 = \text{“}chop\ bread\text{”}, s_2 = \text{“}choose\ condiment\text{”}, \dots$
- Then, the total number of ways that E can be conducted in is

$$\prod_{i=1}^{k} n_i = n_1 \times n_2 \times \cdots \times n_k$$

A Familiar Example

- How many subsets are there of a set of 4 elements?
- Example: $\{a, b, c, d\}$
  - a: in or out. 2 choices.
  - b: in or out. 2 choices.
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- Generalization: there are 2^n subsets of a set of size n.
  - But you already knew this.

Permutations

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- A **permutation** of “machinery” is **a string which results by re-organizing the characters of “machinery” around**.
  - Examples: chyirenma, hcyranemi, machinery (!)
  - Question: How many permutations of “machinery” are there?

Permutations

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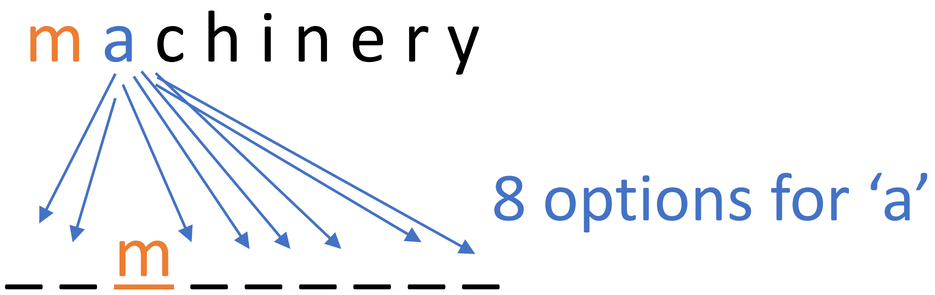

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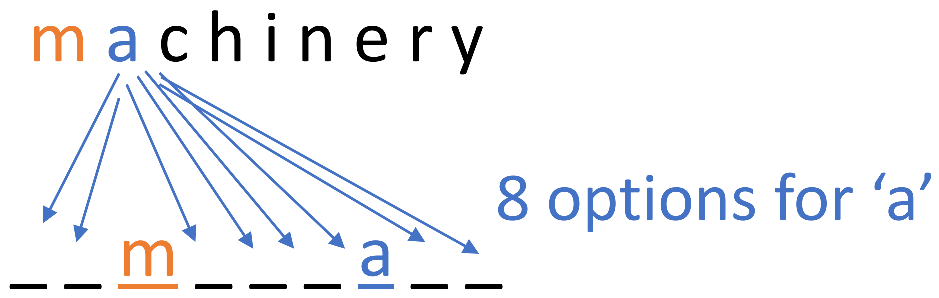

Permutations

m a c h i n e r y

_ _ m _ _ _ _ a _ _

7 options for ‘c’…

Permutations

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_ _ m _ _ c a _ _

Permutations

m a c h i n e r y

6 options for ‘h’…

_ _ m _ _ c a _ _

Permutations

m a c h i n e r y

h _ m _ _ c a _ _

6 options for ‘h’…

Permutations

m a c h i n e r y

5 options for ‘i’

h _ m _ _ c a _ _

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Permutations

m a c h i n e r y

h _ m _ _ c a _ i

4 options for ‘n’

Permutations

m a c h i n e r y

h _ m _ n c a _ i

4 options for ‘n’

Permutations

m a c h i n e r y

h _ m _ n c a _ i

3 options for ‘e’

Permutations

m a c h i n e r y

h e m _ n c a _ i

3 options for 'e'

Permutations

m a c h i n e r y

h e m _ n c a _ i

2 options for ‘r’

Permutations

m a c h i n e r y

2 options for ‘r’

h e m _ n c a r i

Permutations

m a c h i n e r y

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1 option for ‘y’

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Total #possible permutations $= 9 \times 8 \times \cdots \times 2 \times 1 = 9! = 362880$

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That’s a lot! (Original string has length 9)

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In general, for a string of length n we have ourselves $n!$ different permutations!

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  - **What is the answer?**

Thought Experiment

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  - Answer: $\frac{6!}{2}$

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  - Observe all the possible positions of the various 's''s:
    - $s_1 c i s_2 s_3 o r$
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    - $s_2 c i s_1 s_3 o r$
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3! = 6 different ways to arrange those 3 's's

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- Therefore, the total #permutations when not assume different 's's is

$$\frac{7!}{3!} = \frac{1 \times \cancel{2 \times 3} \times 4 \times 5 \times 6 \times 7}{1 \cancel{\times 2 \times 3}} = 20 \times 42 = 840$$

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| 6 | 12 |
| 16 | Something Else |

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$4! = 24$ different ways.

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- Key: <u>**for every one**</u> of these two (equivalent) permutations, we have 4! equivalent permutations because of the ‘o’s! <u>***(MULTIPLICATION RULE)***</u>

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- Key: **for every one** of these two (equivalent) permutations, we have 4! equivalent permutations because of the ‘o’s! ***(MULTIPLICATION RULE)***
- Final answer:

$$\#permutations = \frac{12!}{4! \cdot 2!} = \frac{5 \cdot 6 \cdot \ldots \cdot 11 \cdot 12}{2} = 5 \cdot 6^2 \cdot \ldots \cdot 10 \cdot 11 = 9{,}979{,}200$$

Important "Pedagogical" Note

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- In the previous problem, we came up with the quantity

$$\frac{12!}{4! \cdot 2!} = 9{,}979{,}200$$

- **How you should answer in an exam:** $\frac{12!}{4! \cdot 2!}$
- **Don't perform computations, like** $9{,}979{,}200$
  - Helps **you** save time and **us when grading** ☺

For You!

- Consider the word “bookkeeper” (according to this website, the only unhyphenated word in English with three consecutive repeated letters)

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$$\frac{10!}{2! \cdot 2! \cdot 3!}$$

Don’t forget the third ‘e’!

More Practice

- What about the #non-equivalent permutations for the word

combinatorics

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combinatorics

$$\frac{13!}{2! \cdot 2! \cdot 2!} = \cdots$$

General Template

- Total # permutations of a string σ of letters of length *n* where there are n_a $'a's, n_b$ $'b's, n_c$ $'c's, \ldots n_z$ 'z's

$$\frac{n!}{n_a! \times n_b! \times \cdots \times n_z!}$$

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- Claim: This formula is problematic when some letter (a, b, …, z) is **not** contained in σ

Yes

No

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Remember:
0! = 1

Yes

No

r-permutations

- Warning: permutations (as we've talked about them) are best presented with strings.

r-permutations

- Warning: permutations (as we've talked about them) are best presented with strings.
- r-permutations: Those are best presented with sets.
  - Note that $r \in \mathbb{N}$
  - So we can have 2-permutations, 3-permutations, etc

r-permutations: Example

- I have ten people.

- My goal: pick three people for a picture, where **order of the people matters.**

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- Examples: shortest-to-tallest or tallest-to-shortest or something-in-between

r-permutations: Example

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- My goal: pick three people for a picture, where **order of the people matters.**
- Examples: Jenny-Fred-Bob or Fred-Jenny-Bob or Fred-Bob-Jenny

r-permutations: Example

- I have ten people.

- My goal: pick three people for a picture, where **order of the people matters.**
- In how many ways can I pick these people?

r-permutations: Example

r-permutations: Example

I need three people for this photo. You guys figure out your order.

r-permutations: Example

10 ways to pick the first person…

. . .

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r-permutations: Example

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___ ___ ___

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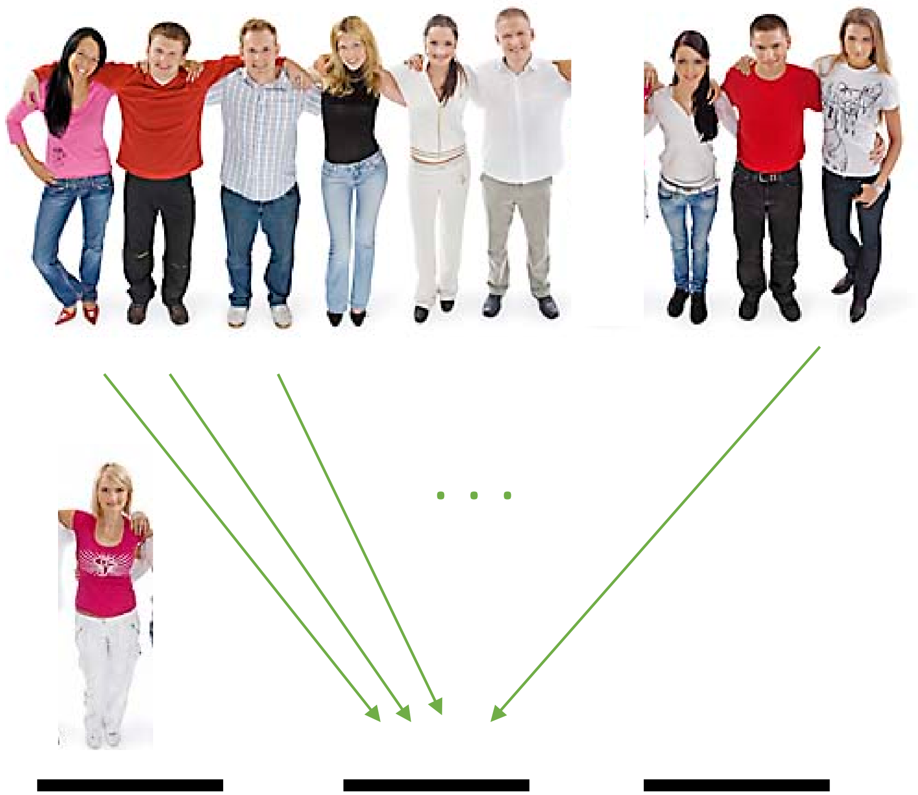

9 ways to pick the **second** person…

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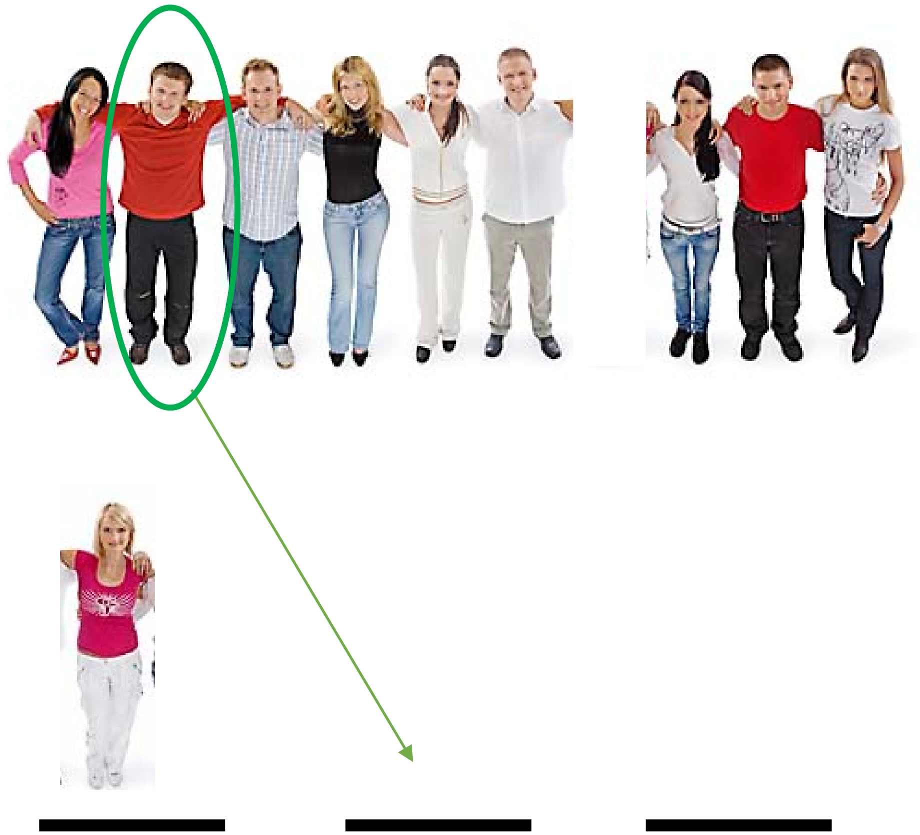

9 ways to pick the **second** person…

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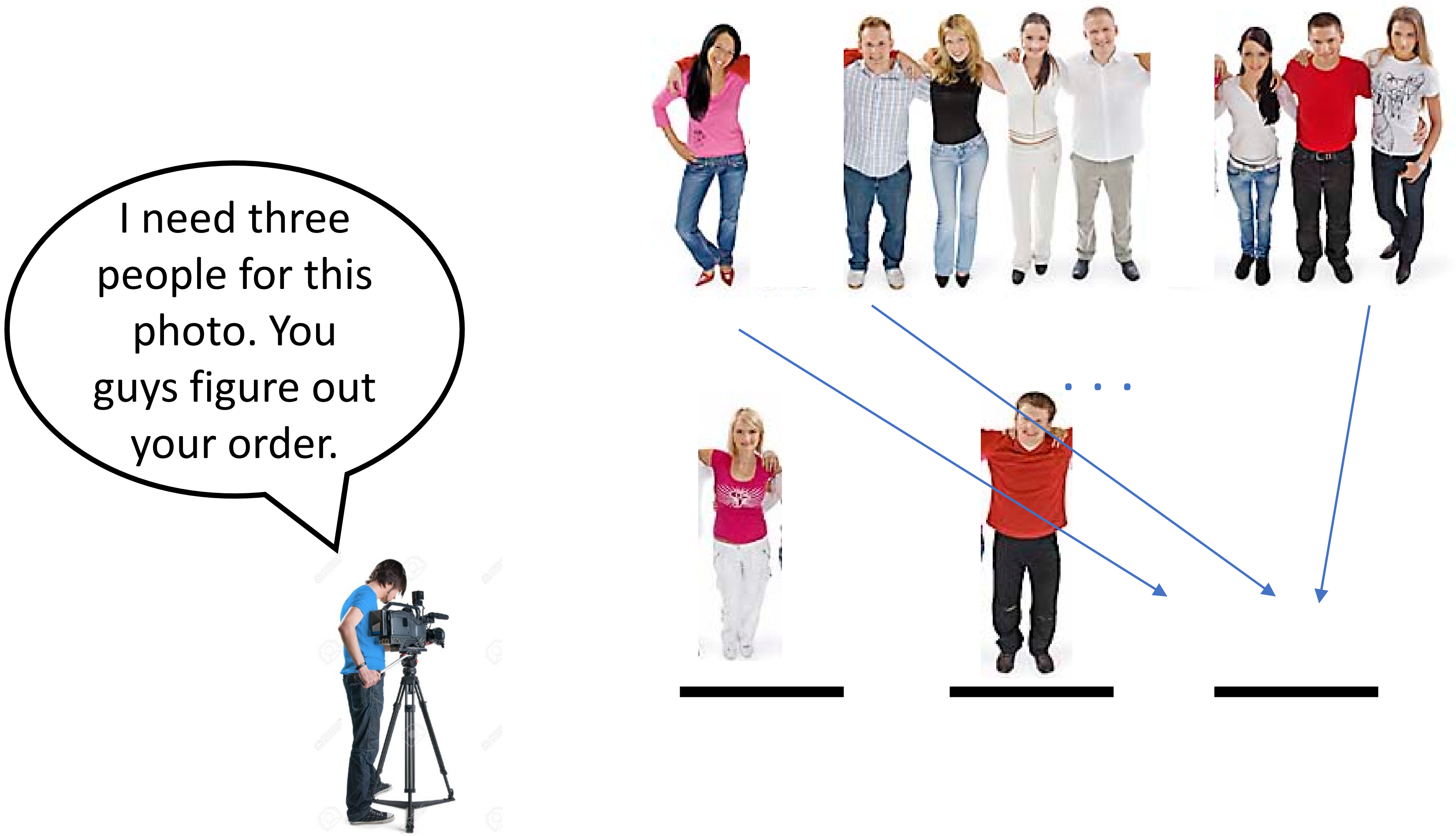

8 ways to pick the **third** person…

r-permutations: Example

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I need three people for this photo. You guys figure out your order.

r-permutations: Example

8 ways to pick the **third** person…

r-permutations: Example

For a total of 10 $\times$ 9 $\times$ 8 =720 ways.

r-permutations: Example

I need three people for this photo. You guys figure out your order.

For a total of $10 \times 9 \times 8 = 720$ ways.

Note: $10 \times 9 \times 8 = \frac{10!}{(10-3)!}$

Example on Books

- Clyde has the following books on his bookshelf
  - Epp, Rosen, Hughes, Bogart, Davis, Shaffer, Sellers, Scott

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- Clyde has the following books on his bookshelf
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$$\frac{8!}{(8-5)!} = \frac{8!}{3!}$$

General Formula

- Let $n, r \in \mathbb{N}$ such that $0 \leq r \leq n$. The total ways in which we can select r elements from a set of n elements where order matters is equal to:

$$P(n, r) = \frac{n!}{(n-r)!}$$

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"P" for permutation. This quantity is known as the r-permutations of a set with n elements.

Pop Quizzes

1) $P(n, 1) = \cdots$

0	1	n	$n!$

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- Two ways to convince yourselves:
  - Formula: $\frac{n!}{(n-1)!} = n$
  - Semantics of r-permutations: In how many ways can I pick 1 element from a set of n elements? Clearly, I can pick any one of n elements, so n ways.

Pop Quizzes

2) $P(n, n) = \cdots$

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Pop Quizzes

2) $P(n, n) = \cdots$ 0 | 1 | n | $n!$

- Again, two ways to convince ourselves:
  - Formula: $\frac{n!}{(n-n)!} = \frac{n!}{0!}$
  - Semantics: $n!$ ways to pick all of the elements of a set and put them in order!

Pop Quizzes

3) $P(n, 0) = \cdots$

0	1	n	$n!$

Pop Quizzes

3) $P(n, 0) = \cdots$

0	1	n	$n!$

- Again, two ways to convince ourselves:
  - Formula: $\frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$
  - Semantics: Only one way to pick nothing: just pick nothing and leave!

Practice

1. How many MD license plates are possible to create?

Practice

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Practice

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Practice

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Remember these phrases!

Combinations (that “n choose r” stuff)

- Earlier, we discussed this example:

- Our goal was to pick three people for a picture, where **order of the people mattered.**

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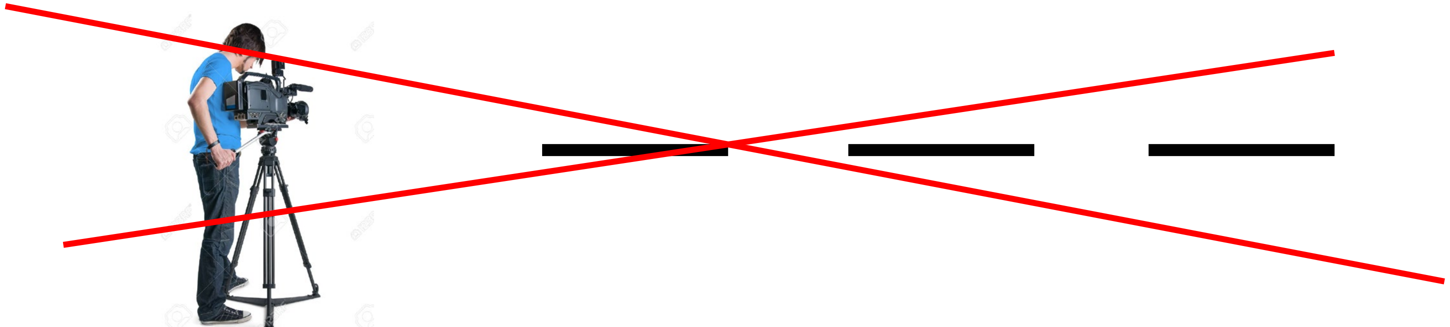

- We now change this setup to forming a PhD defense committee (also 3 people).
- In this setup, does order matter?

Combinations (that “n choose r” stuff)

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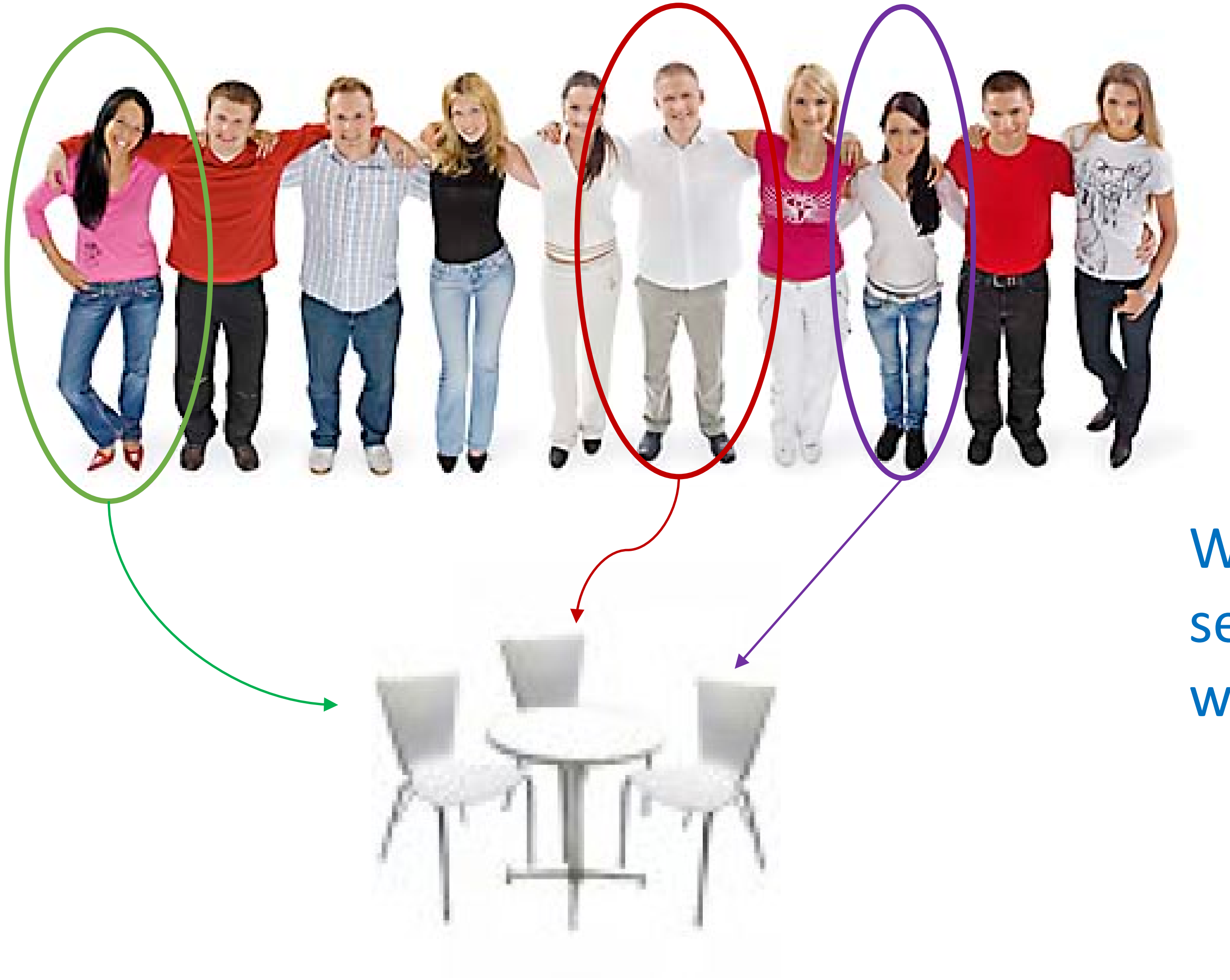
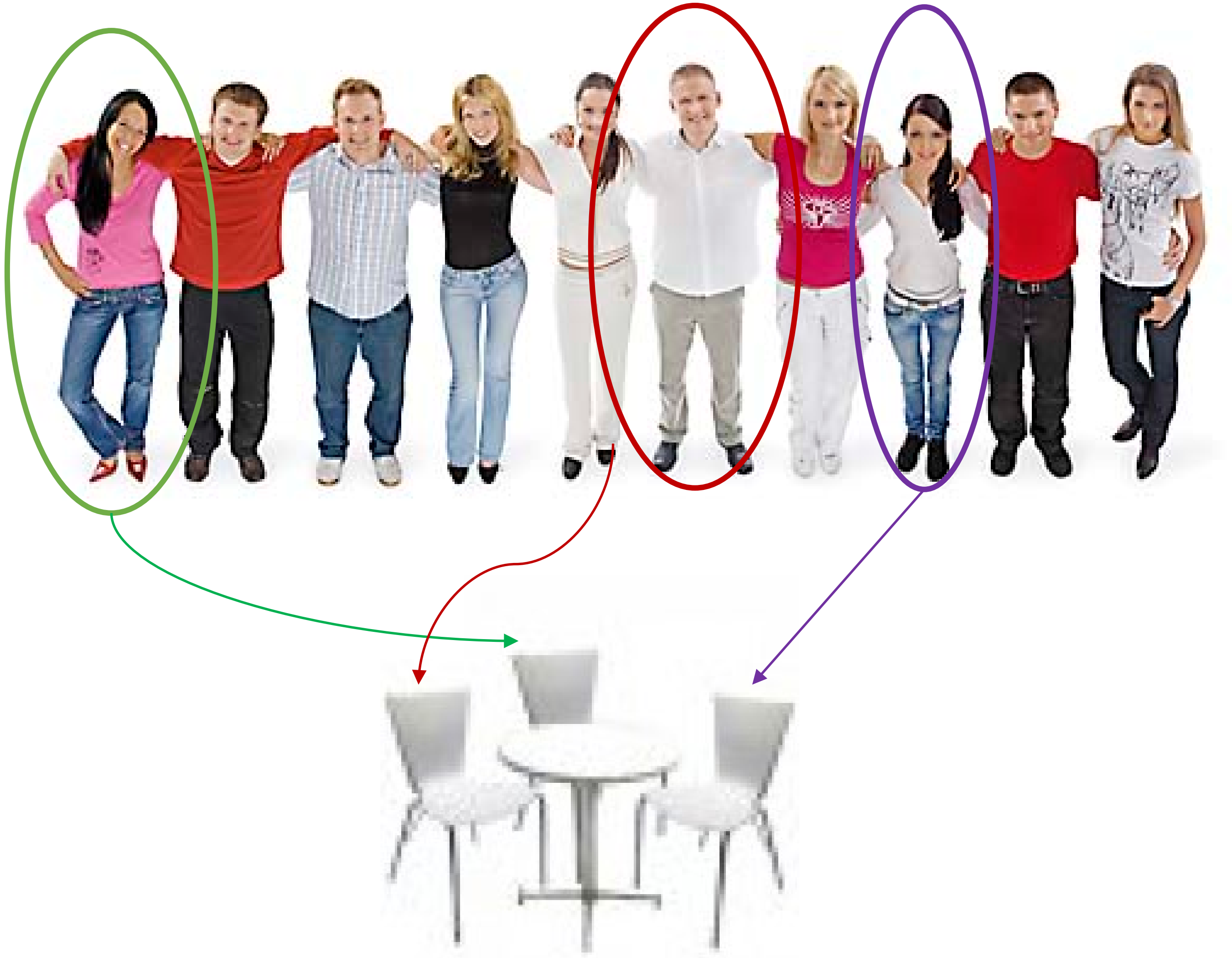


We can make this selection in $P(10, 3)$ ways…

Combinations (that "n choose r" stuff)



We can make this selection in $P(10,3)$ ways… but **since order doesn't matter**, we have 3! permutations of these people that are equivalent.

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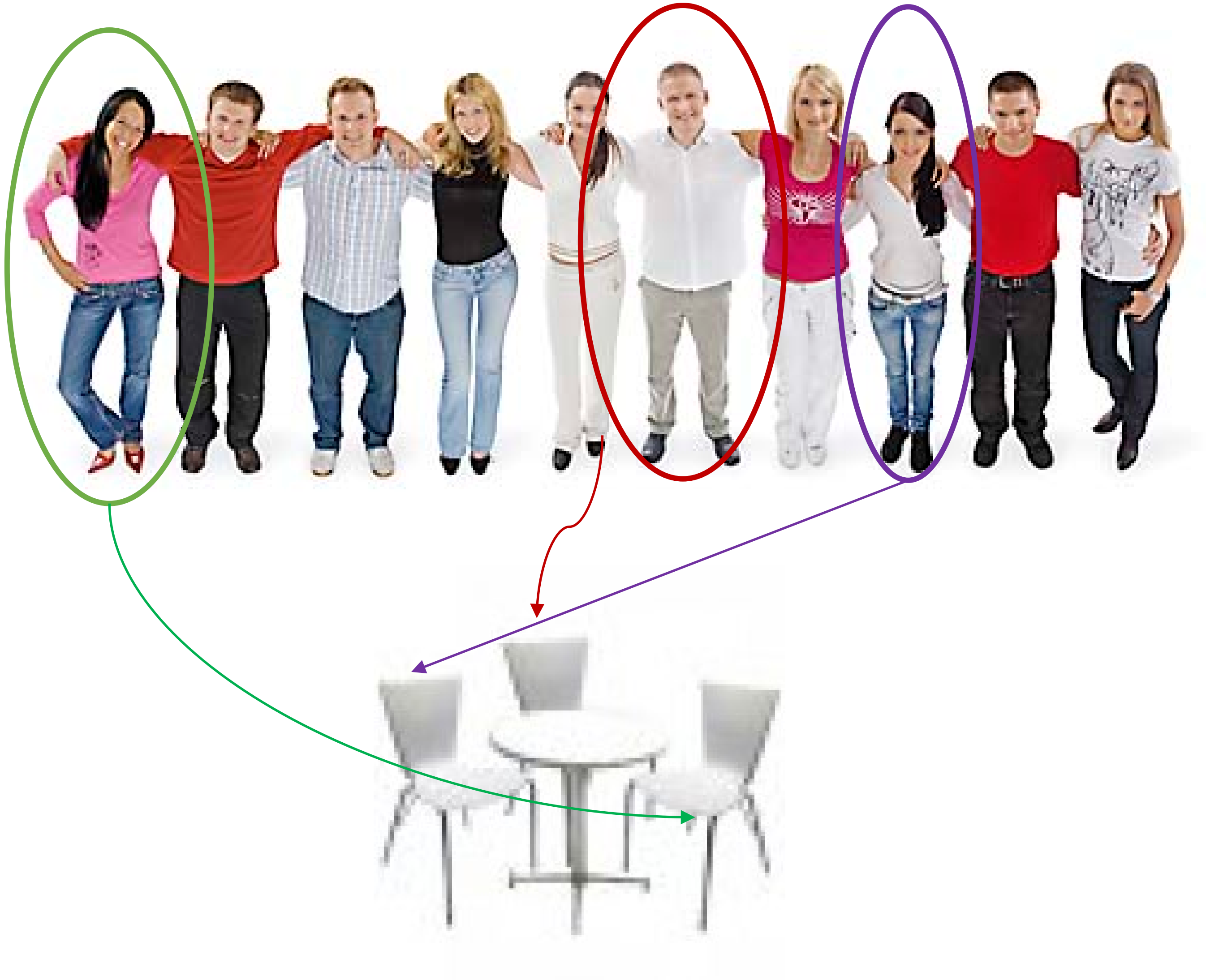

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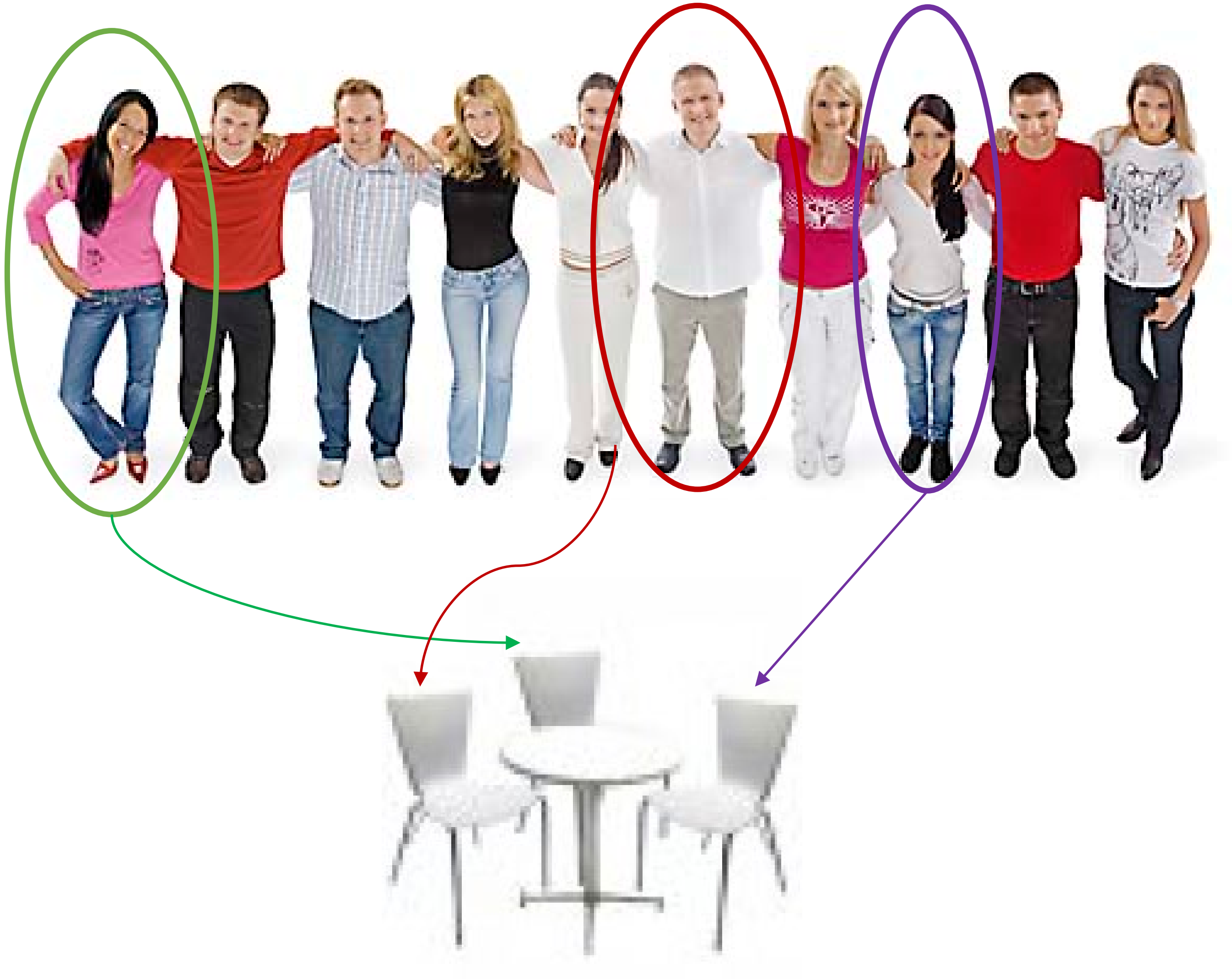

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Overcount ☹

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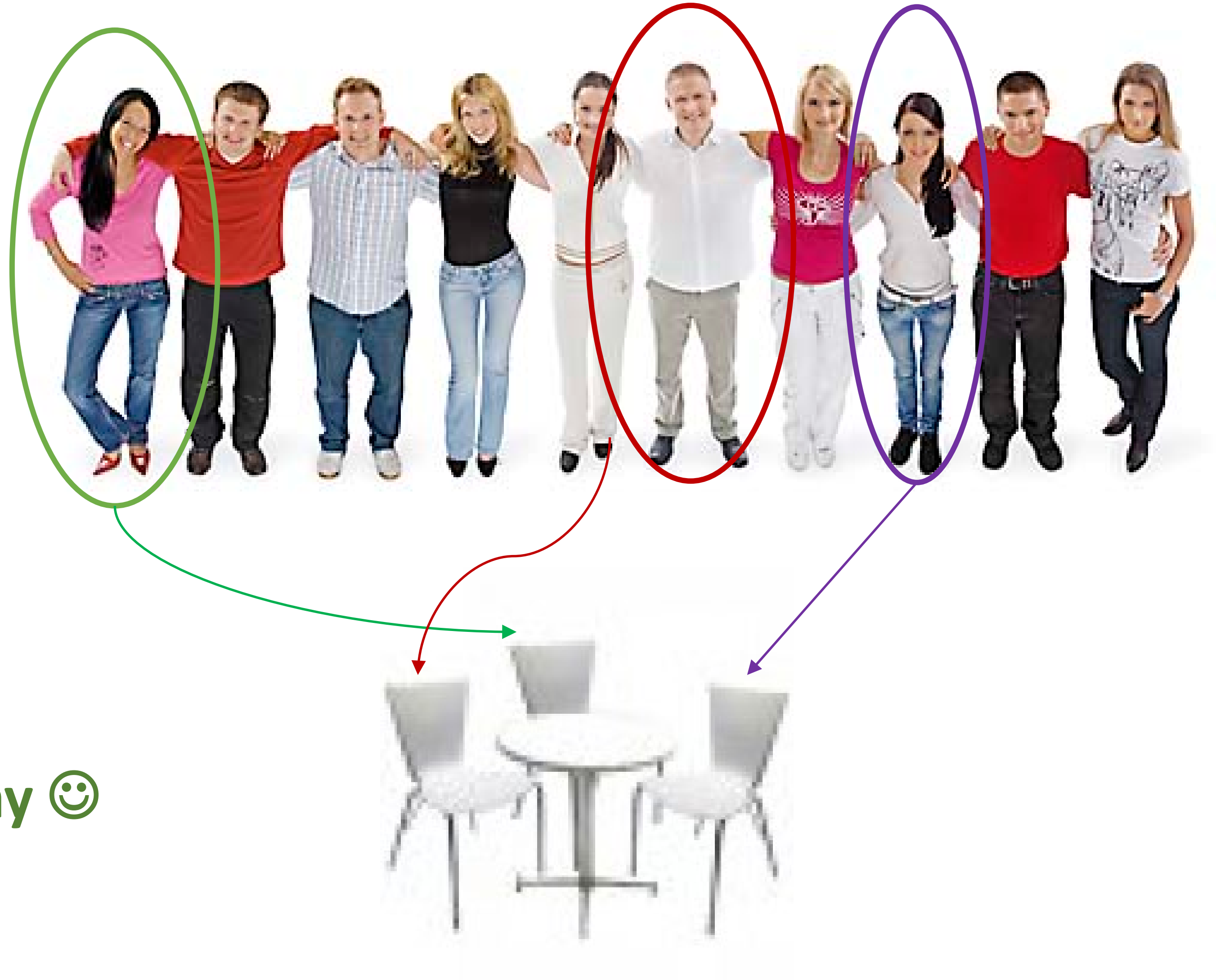

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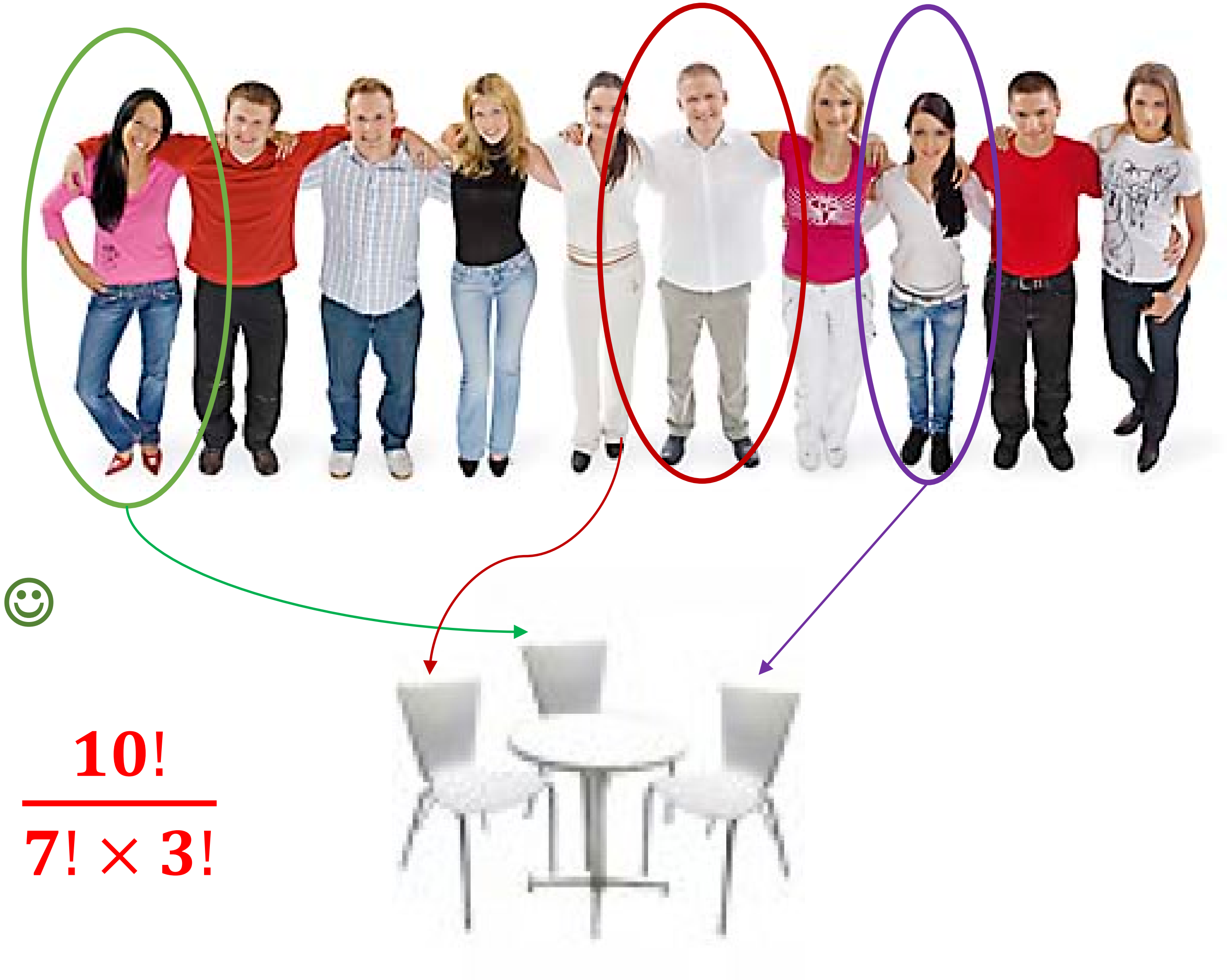

Overcount ☹

In a precise way ☺

$$\frac{P(10,3)}{3!} = \frac{10!}{7! \times 3!}$$

We can make this selection in $P(10,3)$ ways… but **since order doesn't matter**, we have 3! permutations of these people that are equivalent.

Closer Analysis of Example

- Note that essentially we are asking you: Out of a set of 10 people, how many subsets of 3 people can I retrieve?

$\binom{n}{r}$ Notation

- The quantity

$$\frac{\boldsymbol{P(10,3)}}{\boldsymbol{3!}}$$

is the number of *3-combinations* from a set of size 10, denoted thus:

$$\binom{n}{3}$$

and pronounced "n choose 3".

$\binom{n}{r}$ Notation

- Let $n, r \in \mathbb{N}$ with $0 \leq r \leq n$
- Given a set A of size n, the total number of subsets of A of size r is:

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

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- Pop quiz: $(\forall n, r \in \mathbb{N})[(0 \leq r \leq n) \Rightarrow (\binom{n}{r} \leq P(n,r))]$

True

False

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- Pop quiz: $(\forall n, r \in \mathbb{N})[(0 \leq r \leq n) \Rightarrow (\binom{n}{r} \leq P(n,r))]$

Recall that

$\binom{n}{r} = \frac{P(n,r)}{r!}$ and $r! \geq 1$

True

False

Quiz

Quiz

1	n	$n!$	Sth else

1. $\binom{n}{1} =$

Quiz

1	n	$n!$	Sth else

1. $\binom{n}{1} = n$

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Quiz

1	n	$n!$	Sth else

1. $\binom{n}{1} = n$
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STOP RECORDING