

More Induction Problems CMSC 250

1. Prove $21 \mid (4^{n+1} + 5^{2n-1})$ for every positive integer n .

2. Prove that for every positive integer n ,

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1)$$

3. Given

$$a_n = \begin{cases} 1 & n = 1 \\ 3 & n = 2 \\ a_{n-2} + 2a_{n-1} & n \geq 3 \end{cases}$$

Prove that a_n is odd for all integers $n \geq 1$.

4. Given

$$a_n = \begin{cases} 1 & n = 1 \\ 2 & n = 2 \\ \sum_{i=1}^{n-1} (i-1)a_i & n \geq 3 \end{cases}$$

Prove that $a_n = (n-1)!$ for all integers $n \geq 3$.

5. Given

$$a_n = \begin{cases} 1 & n = 1 \\ 2 & n = 2 \\ \frac{a_{n-1}}{a_{n-2}} & n \geq 3 \end{cases}$$

(a) Prove that

$$a_n = \begin{cases} 1 & \text{if } n \equiv 1, 4 \pmod{6} \\ 2 & \text{if } n \equiv 2, 3 \pmod{6} \\ \frac{1}{2} & \text{if } n \equiv 0, 5 \pmod{6} \end{cases}$$

for all positive integers n .

(b) Prove that for all nonnegative integers j , $\sum_{i=1}^6 a_{j+i} = 7$

6. Use Constructive Induction to find constants A, B, C for

$$\sum_{i=1}^n 4i - 3 = An^2 + Bn + C.$$

7. Use Constructive Induction to find constants A, B, C, D for

$$\sum_{i=1}^n i(i+2) = An^3 + Bn^2 + Cn + D.$$

8. Use Constructive Induction to find constants A, B, C for

$$a_n = \begin{cases} 1 & n = 1 \\ 4 & n = 2 \\ 9 & n = 3 \\ a_{n-1} - a_{n-2} + a_{n-3} + 2(2n - 3) & n \geq 4 \end{cases}$$

such that $a_n = An^2 + Bn + C$.

9. Use Constructive Induction to a constant A bound for

$$\sum_{i=1}^n \frac{1}{(i+2)(i+3)}$$

such that $a_n \leq An$