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In any SAT assignment need $x_1 = T$ and $x_3 = F$ so $\neg x_1 \vee x_3$ is F.
Hence NOT in SAT.

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UNKNOWN TO SCIENCE If there are no restrictions on the formula, then unknown if there is an algorithm better than $\sim 2^n$.

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Notation We denote Polynomial Time by **P**.

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UNKNOWN TO SCIENCE In fact, The $(1.306)^n$ algorithm is the best algorithm we know.

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Upshot Determining if 3SAT is in P is a hard problem.

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4. For all $\alpha < 2$
5. 2

Answer $2\cos(\pi/7)$. I'm surprised too! Used hard math.
Is there hope to improve this? VOTE on which is known:

1. Known techniques you can't get $\alpha \geq 2$.
2. Known techniques you can't get $\alpha \geq 2\cos(\pi/8) \sim 1.848$.
3. Known techniques you can't get $\alpha > 2\cos(\pi/7) \sim 1.848$.

Answer $2\cos(\pi/7)$. So new ideas are needed.

Upshot Determining if 3SAT is in P is a hard problem.

How Long Has It Been Open? Posed in 1971. Sort of.

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- The complexity of 3-SAT is **important** since it relates to the complexity of many other problems.
- Many of the problems 3-SAT is equivalent to have been worked on for 90 or more years; hence, it is unlikely they are in P. Hence it is unlikely that 3-SAT is in P.

Proper Terminology and What Do People In the Know Think?

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Scenario Your boss wants you to solve the TSP problem. You know that finding the **optimal** solution is likely not easy to do. So you know to look for an **approximation**. Perhaps something that is at worst twice optimal.

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More generally, if you know a problem is equivalent to SAT then you know that you should not look for an optimal poly time solutions. There are many other options to try.

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Jace can go to 8196, which is further than I can go.