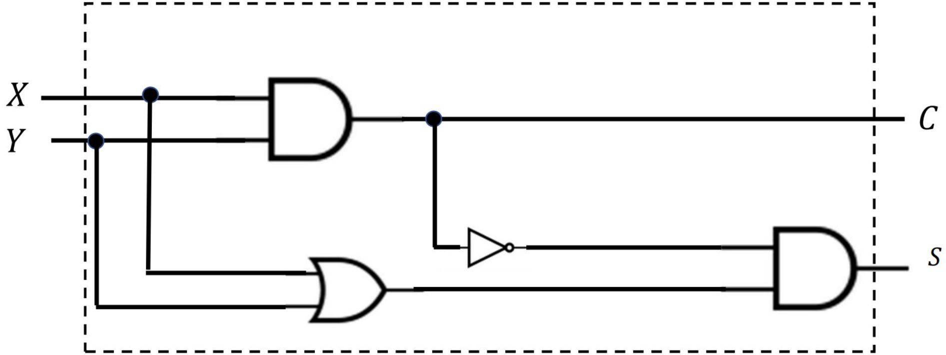
Circuits

250H

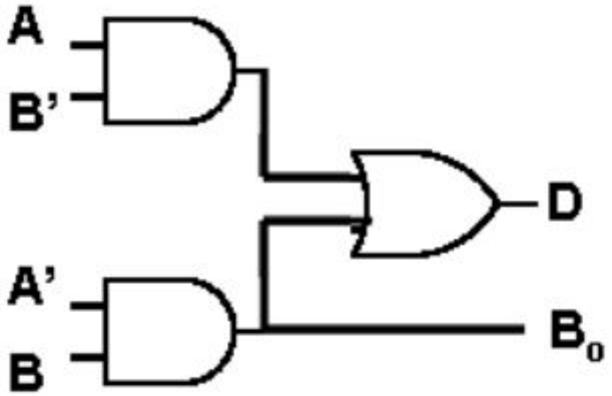
Half Adders

X	Y	C	S
0	0	**0**	**0**
0	1	**0**	**1**
1	0	**0**	**1**
1	1	**1**	**0**



Half Subtractors

X	Y	D	B
0	0	**0**	**0**
0	1	**1**	**1**
1	0	**1**	**0**
1	1	**0**	**0**



Exclusive OR

X	Y	$X \oplus Y$
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

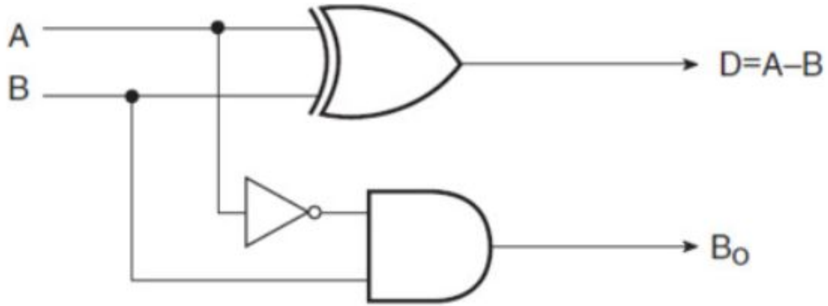
Exclusive OR

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

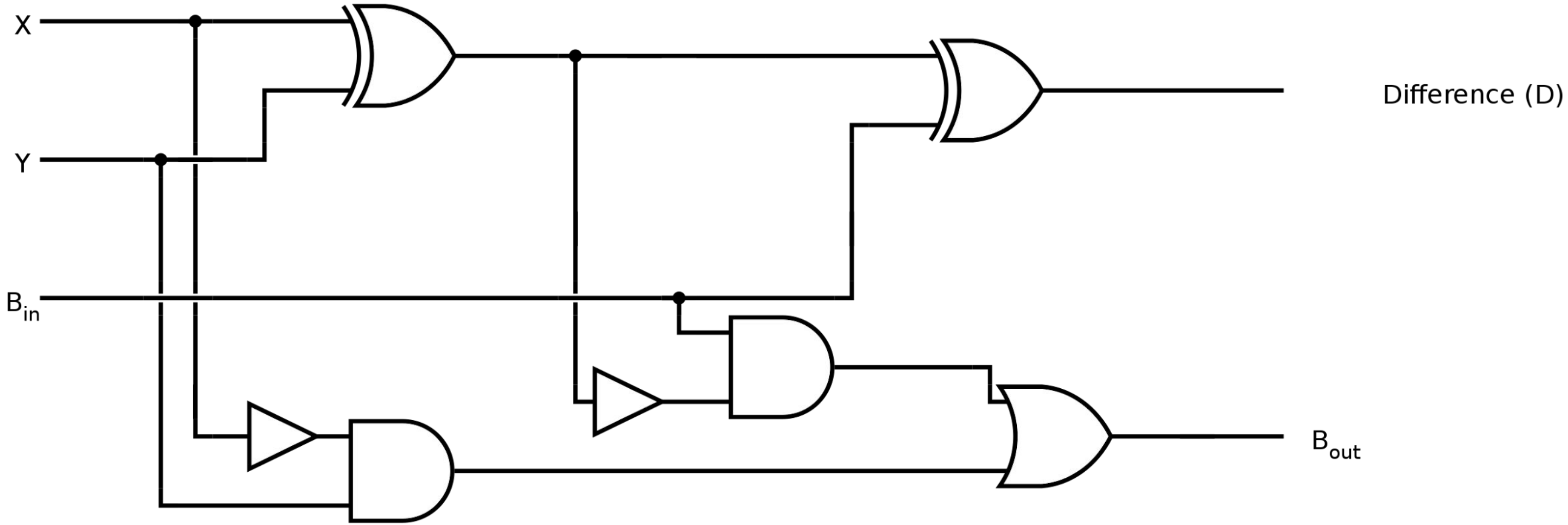
Wait didn’t we just see that column

Half Subtractors

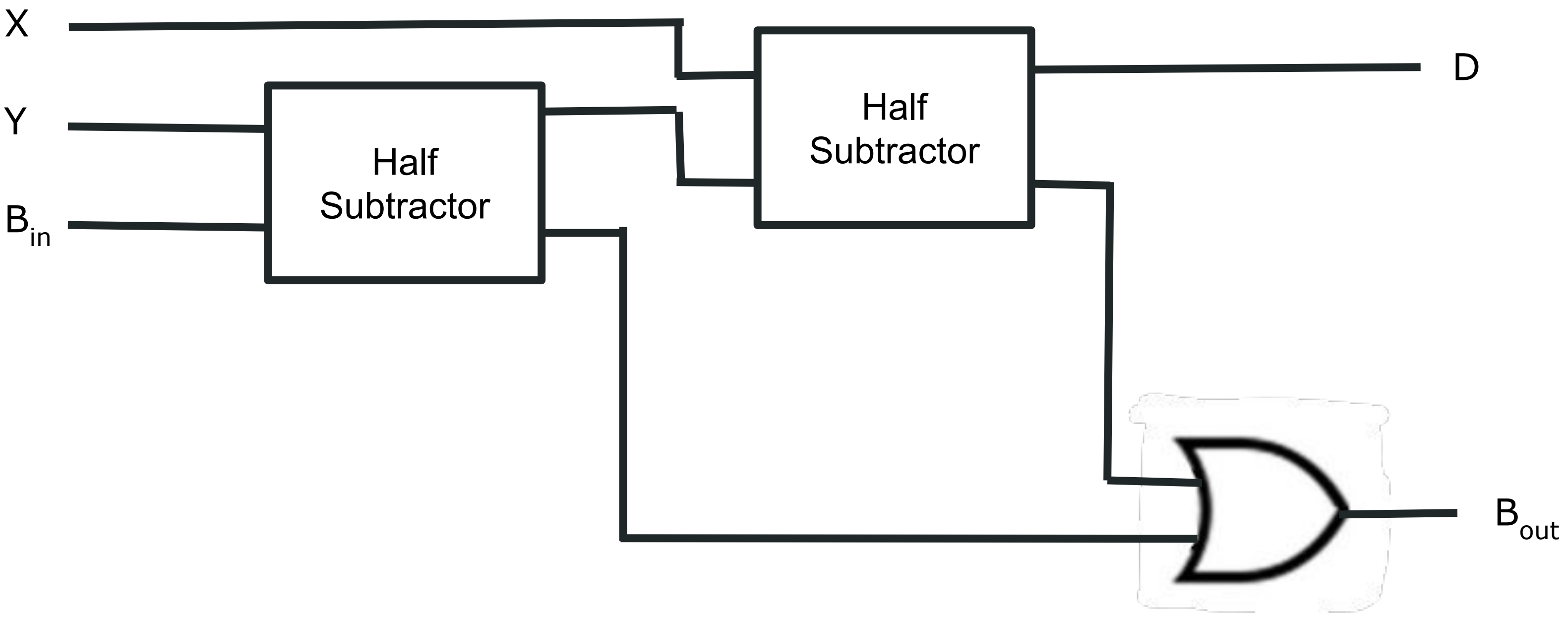
X	Y	D	B
0	0	**0**	**0**
0	1	**1**	**1**
1	0	**1**	**0**
1	1	**0**	**0**



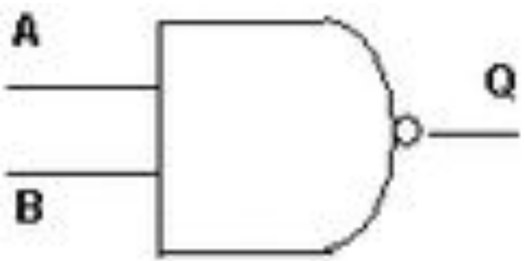
Full Subtractors



Full Subtractors

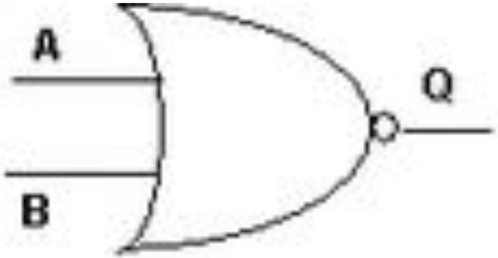


NAND and NOR



NAND

A	B	Q
0	0	1
0	1	1
1	0	1
1	1	0



NOR

A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

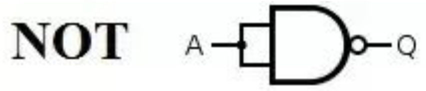
NAND and NOR

- Any circuit can be created with only **NAND** and **NOR** gates

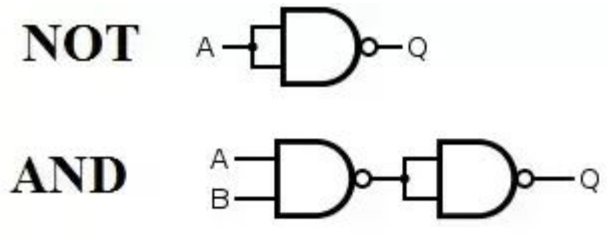
NAND and NOR

- Any circuit can be created with only **NAND** and **NOR** gates
- Try and create **NOT**, **AND**, and **OR** using only **NAND** gates

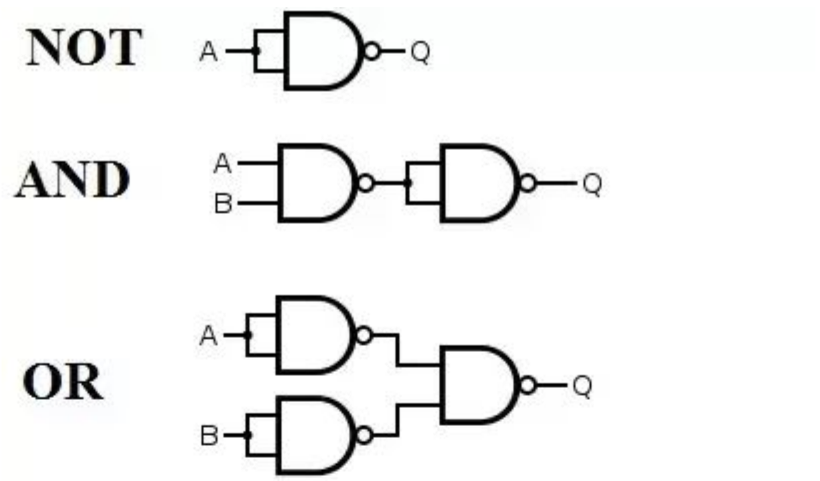
Building Gates with NAND



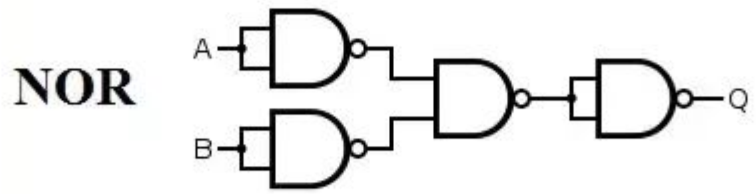
Building Gates with NAND



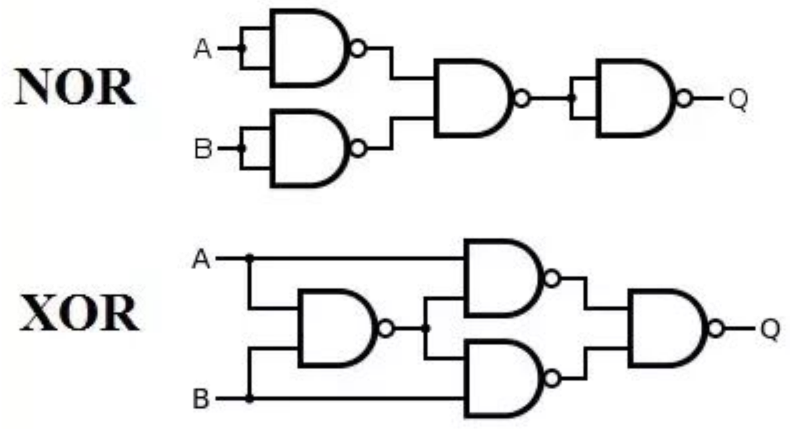
Building Gates with NAND



Building Gates with NAND



Building Gates with NAND



Boolean Algebra Identities

TABLE 3
Basic Identities of Boolean Algebra

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

Prove using Algebraic Manipulation

1.	$X + 0 = X$	2.	$X \cdot 1 = X$	
3.	$X + 1 = 1$	4.	$X \cdot 0 = 0$	
5.	$X + X = X$	6.	$X \cdot X = X$	
7.	$X + \overline{X} = 1$	8.	$X \cdot \overline{X} = 0$	
9.	$\overline{\overline{X}} = X$			
10.	$X + Y = Y + X$	11.	$XY = YX$	Commutative
12.	$X + (Y + Z) = (X + Y) + Z$	13.	$X(YZ) = (XY)Z$	Associative
14.	$X(Y + Z) = XY + XZ$	15.	$X + YZ = (X + Y)(X + Z)$	Distributive
16.	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$	17.	$\overline{X \cdot Y} = \overline{X} + \overline{Y}$	DeMorgan's

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

Let's Check with Truth Tables

X	$\overline{X}$	Y	$\overline{Y}$	$\overline{X}\,\overline{Y} + \overline{X}Y + XY$	$\overline{X} + Y$
0	1	0	1	**1**	**1**
0	1	1	0	**1**	**1**
1	0	0	1	**0**	**0**
1	0	1	0	**1**	**1**

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

Prove using Algebraic Manipulation

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

Prove using Algebraic Manipulation

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\bar{Y} + \bar{X}Y + \bar{X}Y + XY = \bar{X} + Y$$

Prove using Algebraic Manipulation

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\bar{Y} + \bar{X}Y + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\left(\bar{Y} + Y\right) + Y\left(\bar{X} + X\right) = \bar{X} + Y$$

Prove using Algebraic Manipulation

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\bar{Y} + \bar{X}Y + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\left(\bar{Y} + Y\right) + Y\left(\bar{X} + X\right) = \bar{X} + Y$$

$$\bar{X}(1) + Y(1) = \bar{X} + Y$$

Prove using Algebraic Manipulation

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\bar{Y} + \bar{X}Y + \bar{X}Y + XY = \bar{X} + Y$$

$$\bar{X}\left(\bar{Y} + Y\right) + Y\left(\bar{X} + X\right) = \bar{X} + Y$$

$$\bar{X}(1) + Y(1) = \bar{X} + Y$$

$$\bar{X} + Y = \bar{X} + Y$$

Let’s Make a Circuit

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

- How many gates on the left?

Let’s Make a Circuit

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

- How many gates on the left?
  - 8

Let’s Make a Circuit

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

- How many gates on the left?
  - 8
- How many gates on the right?

Let’s Make a Circuit

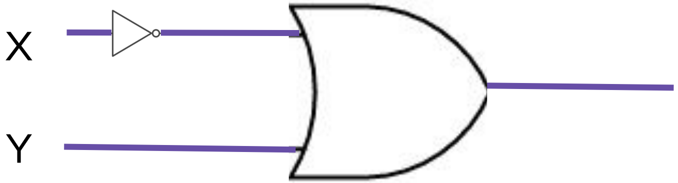
$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

- How many gates on the left?
  - 8
- How many gates on the right?
  - 2

Let’s Make a Circuit

$$\bar{X}\bar{Y} + \bar{X}Y + XY = \bar{X} + Y$$

- How many gates on the left?
  - 8
- How many gates on the right?
  - 2
- 2 is much better than 8



Use Algebraic Manipulation to show the following

1. $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C = 1$
2. $Y + \bar{X}Z + X\bar{Y} = X + Y + Z$
3. $\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y} + XZ + Y\bar{Z}$
4. $AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D = B + \bar{C}D$
5. Given that $A \cdot B = 0$ and $A + B = 1$, Show

$$(A + C) \cdot \left(\bar{A} + B\right) \cdot (B + C) = B \cdot C$$