Domains with a Finite Number of Primes

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Actually, we should think in terms of $\mathbb{Z}$, not $\mathbb{N}$.

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We will address this when we generalize the concept of
an infinite number of primes

What is a Domain?

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Domains with a Finite Number of Primes
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On the next slide we will define **Integral Domain** which is a set of numbers that have many of the same properties as the integers.

Integral Domains

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4. $0, 1 \in \mathbb{D}$.

Note We **did not** require that $(\forall x \in \mathbb{D} - \{0\})[\frac{1}{x} \in \mathbb{D}]$.

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4. $\mathbb{G} = \{a + bi : a, b \in \mathbb{Z}\}$ is an integral domain.
5. $\mathbb{F} = \{a + b\sqrt{-5} : a, b \in \mathbb{Z}\}$ is an integral domain.

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3. $x \in \mathbb{D} - \{0\}$ is **composite** if $(\exists y, z \notin \mathbb{U})[x = yz]$.

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Convention The phrase $\mathbb{D}$ **has an infinite number of primes** means that there $\mathbb{D}$ has an infinite sequence of primes $p_1, p_2, \ldots$ such that
For all i, j $\frac{p_i}{p_j} \notin \mathbb{U}$.

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Note that 2 and -2 would not both be on the list.

Which of $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{G}$, $\mathbb{F}$ have an Inf. Numb. of Primes?

$\mathbb{Z}$ we know has an infinite number of primes.
Discuss Does $\mathbb{Q}$ have an infinite number of primes?

Which of $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{G}$, $\mathbb{F}$ have an Inf. Numb. of Primes?

$\mathbb{Z}$ we know has an infinite number of primes.
Discuss Does $\mathbb{Q}$ have an infinite number of primes?
Every element of $\mathbb{Q} - \{0\}$ is a unit. So there are ZERO primes.

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It does— you may look into that on a later HW.

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Every Integral Domain has either Infinite or Zero primes

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Answer on the next page.

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So only one prime!

Are ZERO, ONE, INFINITY the Only Possibilities?

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TRUE or FALSE or UNKNOWN TO SCIENCE:

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Next Slide has Answer.

$\exists$ an Integral Domain With Exactly k Primes

We describe a domain with 4 primes.

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- $b \not\equiv 0 \pmod 2 \wedge$
- $b \not\equiv 0 \pmod 3 \wedge$
- $b \not\equiv 0 \pmod 5 \wedge$
- $b \not\equiv 0 \pmod 7$

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The only primes are 2,3,5,7. Proof is like to prior domain.

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Simila: Can get an integral domain with exactly k primes.

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Types of integral domains with a finite number of primes:

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I've already done this with my proof that primes are infinite that uses **Fermat's Last Theorem** ($n = 3$ case) and **Schur's Theorem (From Ramsey Theory)**.