Using Unique Factorization to Proof Numbers Irrational

Recap of Unique Factorization

Thm Every $n \in \mathbb{N}$ factors into primes uniquely.

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Thats our contradiction!

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So it looks like 6 factors two different ways.

But need that $2, 3, 1 + \sqrt{-5}, 1 - \sqrt{-5}$ are all primes.

Proving Numbers in $\mathbb{D}$ are Primes

Recall If $\mathbb{D}$ is a domain then there are three kinds of numbers:

1. u is a **unit** if $(\exists u')[uu' = 1]$. Only units of $\mathbb{D}$: $1, -1$.
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N is helpful since it maps elements of $\mathbb{D}$ (which we don't understand) to $\mathbb{N}$ (which we do understand).

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3 is prime: Similar to 2.

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Proof for $1-\sqrt{-5}$ is similar.

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So only look at primes in $\mathbb{N}$.

Is 23 a prime in $\mathbb{D}$?

Next slide

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Find all $a, b, c, d \in \mathbb{N}$ such that $529 = (a^2 + 5b^2)(c^2 + 5d^2)$?
(Easy to compute! Just look for all $0 \leq a, b, c, d \leq \sqrt{529} = 23$.
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3. $a + b\sqrt{-5} \ne -1$ (so $(a, b) \ne (-1, 0)$).

Might Be a HW

1. Write a program that will, given p, determine if p is a prime in $\mathbb{N}$. If it is then determine if it is a prime in $\mathbb{D}$ by seeing if
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2. For all primes in $\mathbb{N}$ that are ≤ 1000 run the above program. Produce a table of prime in $\mathbb{D}$ that are ≤ 1000.
3. Speculate how to fill this in:
p a prime in $\mathbb{N}$ is also a prime in $\mathbb{D}$ iff BLANK.

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Moral of the Story

1. Using UF we obtain a different proof that $\sqrt{7} \notin \mathbb{Q}$. Technique works for other proofs of irrationality.
2. UF is not obvious. Its false for $\mathbb{D}$ so the proof that $\mathbb{Z}$ has UF would need to use properties of $\mathbb{Z}$ that $\mathbb{D}$ does not have. We won't be doing that proof, but you now know that it is worthy of proof.