Homework 09, MORALLY Due April 21

1. (0 points) What is your name.

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2. (35 points)

   Read or re-read the slides on the Horse Numbers. The numbers $H(n)$ will come up in this problem.

   For $n \geq 2$. Let $I(n)$ be the number of ways that n horses, $x_1, \ldots, x_n$, can finish a race (so orderings with equalities allowed) that have $x_1 < x_n$.

   (a) (0 points but you should do it or convince yourself that you could). What is $I(2)$, $I(3)$, $I(4)$. Do $I(4)$ in such a way that it can be generalized to $I(n)$.

   (b) (30 points) Give a recurrence for $I(n)$. It may also involve $H(n)$. For example, it could be

$$I(n) = I(n-1)H(n-1)+2I(n-2)H(n-2)+\cdots+(n-1)I(n-(n-1))H(n-(n-1)).$$

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3. (35 points)

There are 30 people taking Game Theory.

There are 30 people taking Ramsey Theory.

There are 30 people taking Artificial Intelligence (AI).

There are 10 taking Game theory AND Ramsey Theory.

There are 10 taking Game theory AND AI.

There are 10 taking Ramsey Theory AND AI.

There are 5 taking all three classes.

(a) How many students are there?

(b) How many students are taking Game Theory AND Ramsey Theory but NOT AI?

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4. (30 points-10 points each)

   (This problem is based on material you will see Wednesday April 15.)

   Consider the equation: $w + x + y + z = 100$.

   (a) How many solutions (w, x, y, z) are there with $w, x, y, z \geq 0$?

   (b) How many solutions (w, x, y, z) are there with $w, x, y, z \geq 5$?

   (c) How many solutions (w, x, y, z) with $w \geq 1$ and $x \geq 10$?