

Homework 11, MORALLY Due May 5

1. (0 points) What is your name.

GO TO THE NEXT PAGE

2. (30 points) There are two coins.

- One is fair so the probability of H and T are both $\frac{1}{2}$.
- One is biased so the probability of H is $\frac{1}{3}$ and the probability of T is $\frac{2}{3}$.

Bill will take one not-quite-at-random! Here is what Bill does: He picks the fair coin with probability $\frac{9}{10}$ and the biased coin with probability $\frac{1}{10}$.

- (a) (15 points) Bill flips the coins n times and they are all H. What is the the probability that the coin is biased (as a function of n)?
- (b) s (15 points) (You might want to write a program for this one.) We want a table of n and the probability of bias if there are n H's. We want the table to go from $n = 1$ to $n = 20$. The format of the table should be as follows (we only give the first 3 rows and the numbers are made up.). Real numbers should be to 4 places.

| n | the probability of Bias given H^n |
|-----|-------------------------------------|
| 1 | 0.5101 |
| 2 | 0.631 |
| 3 | 0.6799 |

GO TO THE NEXT PAGE

3. (30 points)
- (a) (15 points) Fill in the following sentence and prove it using the Pigeon hole Principle: *Let A be a subset of $\{1, \dots, 1000\}$ such that $|A| = 300$. Then there are at least XXX subsets of A that have the same sum. (Give XXX both as a combinatorial thing like $\left\lceil \frac{2^{100}}{87} \right\rceil$ (thats not the answer!) and as an actual number like 55 (thats not the answer, or if it is then thats an accident).*
- (b) (15 points) Fill in the following sentence and prove it using the Pigeon hole Principle: *Let A be a subset of $\{-500, \dots, 500\}$ such that $|A| = 300$. Then there are at least YYY subsets of A that have the same sum. (Give XXX both as a combinatorial thing like $\left\lceil \frac{2^{100}}{87} \right\rceil$ (thats not the answer!) and as an actual number like 55 (thats not the answer, or if it is then thats an accident).*
- (c) (0 points) Which is bigger XXX or YYY? Is there an intuition for that?

GO TO THE NEXT PAGE

4. (40 points) Let $a, b, c \geq 2$.

Let $R_2(a, b)$ be the least number n so that, no matter how you 2-color (with R,B) the edges of K_n you will either get a RED K_a or a BLUE K_b . (This was called $R(a, b)$ in the lecture.)

Let $R_3(a, b, c)$ be the least number n so that, no matter how you 3-color (with R,B,G) the edges of K_n you will either get a RED K_a or a BLUE K_b or a GREEN K_c .

(a) (20 points) Show that, for all $a, b, \geq 2$, $R_3(a, b, 2) = R_2(a, b)$.

(b) (20 points) Show that, for all $a, b, c \geq 3$,

$$R(a, b, c) \leq R(a, b, c - 1) + R(a, b - 1, c) + R(a - 1, b, c).$$

5. (0 points but its extra credit)

This problem depends on the Muffin Lecture.

(a) Show that $f(22, 13) \leq \frac{21}{52}$. (Uses Half Method)

(b) Show that $f(33, 20) \leq \frac{41}{100}$. (Uses Interval Method)

MAKE YOUR PROOFS READABLE: use interval diagrams.