Project, MORALLY Due May 5

1. (0 points) What is your name.

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2. (15 points) For each of the following either give what I ask you to give or STATE that its impossible and explain why.

   (a) (3 points) Give a formula on 5 variables that has exactly 3 satisfying assignments.

   (b) (4 points) Give a formula on 3 variables that has exactly 5 satisfying assignments.

   (c) (4 points) An $a, b \in \mathbb{N}$, $a, b \geq 1$, such that there is NO formula on a variables that has exactly b satisfying assignments.

   (d) (4 points) An $a, b \in \mathbb{N}$, $a, b \geq 1$, such that there is NO formula on a variables that has exactly b satisfying assignments AND there is NO formula on b variables that has exactly a satisfying assignments.

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3. (15 points) Give a domain $\mathbb{D} \subseteq \mathbb{R}$ such that all of the following hold: (all of the quantifiers range over $\mathbb{D}$.

- $(\exists L)(\forall x)[L \le x]$. (L is for Lowest).
- $(\exists H)(\forall x)[U \ge x]$. (H is for Highest).
- We use H freely in this sentence. They have the meaning above.
  $(\forall x \ne H)(\exists y)x < y \wedge (\forall z)[\neg(x < z < y)]$.
  (For every element $x \ne H$ there is a *successor*.)
- We use L freely in this sentence. They have the meaning above.
  $(\forall x \ne L)(\exists y)y < x \wedge (\forall z)[\neg(y < z < z)]$.
  (For every element $x \ne H$ there is a *predecessor*.)

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4. (15 points) Let a_i be defined as follows:

$a_0 = 40$

$a_1 = 17$

$a_2 = 40$

$(\forall n \geq 3)[a_n = 3a_{n-1} + 2a_{n-2} + a_{n-3} + 7]$.

Use constructive induction to find b, m such that for all n, $a_n \equiv b \pmod{m}$.

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5. (15 points) Use unique factorization to show the following:

Let $n \in \mathbb{N}$, *and* $n \geq 2$. *Let* $n = p_1^{n_1} \cdots p_k^{n_k}$, *where* $p_1, \ldots, p_k$ *are primes. If* $n^{1/7}$ *is rational then* $(\forall 1 \leq i \leq k)[n_i \equiv 0 \pmod{7}]$.

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6. (15 points) Bill makes his Darling lunch. Let $S, F, D, s, f, d \in \mathbb{N}$.

There are S sandwiches, F fruits, and D desserts to pick from. Her lunch will consist of s different sandwich's, f different fruits, and d deserts some of which can be the same.

Example: If $s = 2$, $f = 3$, and $d = 4$ then Darling could have

SANDWICHES: An egg salad sandwich and a PBJ sandwich.

FRUIT: An apple, an orange, and a grape.

DESERT: 2 Poptarts, 1 cookie, and 1 Hershey kiss (from her Bill!).

(a) (5 points) How many ways can Darling have lunch?

(b) (10 points) One of the sandwiches is Peanut butter and BLUEBERRY JAM, which we denote PBJ. NO other sandwich is blueberrybased.

One of the fruits is a BLUEBERRY. NO other fruit is blueberry-based.

One of the deserts is BLUEBERRY PIE. NO other fruit is blueberrybased.

Darling says *I am okay with having 2 of the 3 items be blueberry-based but I DO NOT want all three of them to be blueberry based!*

How many ways can Darling have lunch?

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7. (15 points) (AS OF APRIL 13 I DID NOT TEACH YOU THE MATERIAL NEEDED FOR THIS QUESTION.)

   How many 5-tuples of natural numbers $(x_1, x_2, x_3, x_4, x_5)$ are such that

   $x_1 + x_2 + x_3 + x_4 + x_5 = 1000$

   and $x_1 \geq 1$, $x_2 \geq 2$, ,$x_3 \geq 3$, $x_4 \geq 4$, and $x_5 \geq 5$.

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8. (20 points) (AS OF APRIL 13 I DID NOT TEACH YOU THE MATERIAL NEEDED FOR THIS QUESTION.)

Let $r, s \in \mathbb{N}$.

Martians play poker with cards that each have

a rank in $\{1, \ldots, r\}$, and

a suit in $\{1, \ldots, s\}$.

They play a 6-card hand.

(a) (5 points) What is the probability of getting two three-of-a-kinds? We denote this $p_{3,3}(r, s)$.

(b) (5 points) What is the probability of getting four-of-a-kinds? We denote this $p_4(r, s)$.

(c) (5 points) Does there exist an r, s such that $p_{3,3}(r, s) < p_4(r, s)$. If so then produces such an r, s. If not then say why not.

(d) (5 points) Does there exist an r, s such that $p_{3,3}(r, s) > p_4(r, s)$. If so then produces such an r, s. If not then say why not.