Take Home Part of the Final DUE May 12 at 10:30AM- NO DEAD CAT

1. (0 points) What is your name.

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2. (20 points on the final)

For PART a of this problem we want the answer in terms of binomial coefficients, powers, $\times$, $+$, $-$, division, floor, ceiling. For example, an answer like $\left\lfloor \frac{2^{10}\times 5-10}{\binom{9}{2}\times 88} \right\rfloor$ (this is NOT the answer) would be in the right form, whereas an answer like 286,000 would NOT be in the right form.

And now for the problem!

Let BILL be the following operation on sets $\{w, x, y, z\}$ of four distinct natural numbers:

$\mathrm{BILL}(w, x, y, z) = w^2 + x^2 + y^2 + z^2$.

(a) (Show all work.) Fill in the n in the following sentence and prove the result:

Let $A \subseteq \{1, \ldots, 1000\}$ of size 50. Then there exists n sets

$$A_1, \ldots, A_n$$

such that:

- *For all i, $|A_i| = 10$.*
- $\mathrm{BILL}(A_1) = \mathrm{BILL}(A_2) = \cdots = \mathrm{BILL}(A_n)$.

(b) NOW give me the answer as an actual number.

SOLUTION:
The number of sets of size 10 is $\binom{50}{10}$.
The min BILL is $1^2 + \cdots + 10^2 = 385$.
The max is $1000^2 + 999^2 + +998^2 + 997^2 + 996^2 = 9910285$.
So the number of options for BILL is $\le 9919285 - 384 = 9909901$.
Hence the answer is

$$\left\lceil \frac{\binom{50}{10}}{9909901} \right\rceil = 1037.$$