CMSC 250H Final Spring 2026

1. (0 points) What is your name?

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2. (20 points)

For this problem we want the answer in terms of binomial coefficients, powers, $\times$, $+$, $-$, division. For example, an answer like $\frac{\binom{9}{2}\times 88}{2^{10}\times 5-18}$ (this is NOT the answer) would be in the right form, whereas an answer like 0.254 would NOT be in the right form.

Leo poker uses the following card deck:

The cards have ranks $\{1,\ldots,10\}$ and suits $\{\clubsuit, \spadesuit, \diamondsuit\}$, so there are 30 cards.

Each player gets 3 cards.

A hand is called a *Leo* if all three cards have the same suit.

A hand is called an *Oliver* if there is one $\clubsuit$, one $\spadesuit$, and one $\diamondsuit$.

In both cases its still a *Leo* or an *Oliver* even it is also a straight.

For example

$\{2\clubsuit, 3\clubsuit, 4\clubsuit\}$ is a *Leo*

and

$\{2\clubsuit, 3\spadesuit, 4\diamondsuit\}$ is an *Oliver*.

(a) (10 points) What is the probability of a hand being a Leo?

(b) (10 points) What is the probability of a hand being an Oliver?

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3. (20 points) Use unique factorization to show the following:

Let $n \in \mathsf{N}$*, and* $n \geq 2$*. Let* $n = p_1^{n_1} \cdots p_k^{n_k}$*, where* $p_1, \ldots, p_k$ *are primes. If* $n^{5/7}$ *is rational then* $(\forall 1 \leq i \leq k)[n_i \equiv 0 \pmod 7]$*.*

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4. (20 points)

For this problem we want the answer in terms of binomial coefficients, powers, $\times$, $+$, $-$, division. For example, an answer like $\frac{2^{10}\times 5-18}{\binom{9}{2}\times 88}$ (this is NOT the answer) would be in the right form, whereas an answer like 286,000 would NOT be in the right form.

(a) Let
$\phi(x_1,\ldots,x_{25}) = x_1 \vee x_2 \vee \cdots \vee x_{25}$.
How many satisfying assignments does ϕ have? No explanation needed.

(b) Let
$\psi(x_1,\ldots,x_{25}) =$
$(x_1 \vee x_2 \vee x_3 \vee x_4 \vee x_5)\wedge$
$(x_6 \vee x_7 \vee x_8 \vee x_9 \vee x_{10})\wedge$
$(x_{11} \vee x_{12} \vee x_{13} \vee x_{14} \vee x_{15})\wedge$
$(x_{16} \vee x_{17} \vee x_{18} \vee x_{19} \vee x_{20})\wedge$
$(x_{21} \vee x_{22} \vee x_{2324} \vee x_{25})$
How many satisfying assignments does ψ have? No explanation needed.

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5. (20 points) Let A, B be constants that you will determine later.

   Let a_i be defined as follows:

   $a_0 = 3$

   $a_1 = 24$

   $(\forall n \ge 3)[a_n = Aa_{n-1} + Ba_{n-2} + 2]$.

   Use constructive induction to find A, B such that

   - $1 \le A, B \le 10$.
   - $A \ne B$
   - $(\forall n \ge 0[a_n \equiv 3 \pmod 7)]$.

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