Homework 07, MORALLY Due April 7

1. (0 points) What is your name.

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2. (40 points)

(a) (0 points) (You may want to write a program to get data.)
(a, b, c) means the NIM game where you can remove a or b or c stones.
Below the condition XXX will be of the form

$n \equiv a_1, a_2, \ldots, a_k \pmod{m}$.

(b) (5 points) Fill in the XXX:
In the game $(1, 3, 5)$*, Player II wins when the game begins with* n *stones iff* $XXX(n)$.

(c) (5 points) Fill in the XXX:
In the game $(1, 4, 6)$*, Player II wins when the game begins with* n *stones iff* $XXX(n)$.

(d) (5 points) Fill in the XXX:
In the game $(1, 5, 7)$*, Player II wins when the game begins with* n *stones iff* $XXX(n)$.

(e) (5 points) Fill in the XXX:
In the game $(1, 6, 8)$*, Player II wins when the game begins with* n *stones iff* $XXX(n)$.

(f) (20 points) Fill in the XXX
In the game $(1, m, m + 2)$*, Player II wins when the game begins with* n *stones iff* $XXX(n, m)$.

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3. Let $a \in \mathbb{N}$. Consider the recurrence

   $T_a(0) = 10$

   $T_a(n) = 2T(\lfloor \frac{n}{2} \rfloor) + an.$

   (a) (0 points) Write a program that will, given a, output $T_a(0)$, $T_a(1)$, ..., $T_a(1000)$.

   (b) (0 points) Run the program for $1 \leq a \leq 20$.

   (c) (30 points) Using your data make a conjecture along the lines of: $T_a(n)$ is roughly $XXX(a, n)$
   (For example, $T_a(n) = a^2n^3$ which is NOT the answer.)

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4. (30 points) Show that, for all $n \in \mathbb{N}$, if $a \geq 2^n$ then DUP wins the DUP-SPOILER game with $(\mathbb{N} + \mathbb{N}, L_a; n)$.