Homework 09, MORALLY Due May 5

1. (0 points) What is your name.

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2. (30 points) There are two coins.

    - One is fair so prob of H and T are both $\frac{1}{2}$.
    - One is biased so prob of H is $\frac{1}{3}$ and Prob of T is $\frac{2}{3}$.

    Bill will take one not-quite-at-random! Here is what Bill does: He picks the fair coin with prob $\frac{9}{10}$ and the biased coin with prob $\frac{1}{10}$.

    (a) (15 points) Bill flips the coins n times and they are all H. What is the prob that the coin is biased (as a function of n)?

    (b) (15 points) (You might want to write a program for this one.) We want a table of n and the prob of bias if there are n H's. We want the table to go from $n = 1$ to $n = 20$. The format of the table should be as follows (we only give the first 3 rows and the numbers are made up.). Real numbers should be to 4 places.

n	Prob of Bias given H^n
1	0.51
2	0.63
3	0.67

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3. (30 points) Fill in the following sentence and prove it using the Pigeon hole Principle:

   In this problem we assume $n \geq 100$.

   (a) (15 points) Fill in the following sentence and prove it using the Pigeon hole Principle: *Let A be a subset of* $\{1, \ldots, 1000\}$ *such that* $|A| = 300$. *Then there are at least XXX subsets of A that have the same sum. (Give XXX both as a combinatorial thing like* $\left\lceil \frac{2^{100}}{87} \right\rceil$ *(thats not the answer!) and as an actual number like 55 (thats not the answer, or if it is then thats an accident).*

   (b) (15 points) Fill in the following sentence and prove it using the Pigeon hole Principle: *Let A be a subset of* $\{-500, \ldots, 500\}$ *such that* $|A| = 300$. *Then there are at least YYY subsets of A that have the same sum. (Give XXX both as a combinatorial thing like* $\left\lceil \frac{2^{100}}{87} \right\rceil$ *(thats not the answer!) and as an actual number like 55 (thats not the answer, or if it is then thats an accident).*

   (c) (0 points) Which is bigger XXX or YYY? Is their an intuition for that?

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4. (40 points) Let $a, b, c \geq 2$.

   Let $R_2(a, b)$ be the least number n so that, no matter how you 2-color (with R,B) the edges of K_n you will either get a RED K_a or a BLUE K_b. (This was called $R(a, b)$ in the lecture.)

   Let $R_3(a, b, c)$ be the least number n so that, no matter how you 3-color (with R,B,G) the edges of K_n you will either get a RED K_a or a BLUE K_b or a GREEN K_c.

   (a) (20 points) Show that, for all $a, b, \geq 2$, $R_3(a, b, 2) = R_2(a, b)$.

   (b) (20 points) Show that, for all $a, b, c \geq 3$,
   $R(a, b, c) \leq R(a, b, c - 1) + R(a, b - 1, c) + R(a - 1, b, c)$.