

Project, MORALLY Due May 5

1. (0 points) What is your name.

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2. (10 points) For each of the following either give what I ask you to give or STATE that its impossible and explain why.
- (a) Give a formula on 5 variables that has exactly 3 satisfying assignments.
 - (b) Give a formula on 3 variables that has exactly 5 satisfying assignments.
 - (c) An a, b such that there is NO formula on a variables that has exactly b satisfying assignments.
 - (d) An a, b such that there is NO formula on a variables that has exactly b satisfying assignments AND there is NO formula on b variables that has exactly a satisfying assignments.

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3. (10 points) Give a domain $\mathbb{D} \subseteq \mathbb{R}$ such that all of the following hold:
(all of the quantifiers range over \mathbb{D} .)

- $(\exists L)(\forall x)[L \leq x]$. (L is for Lowest).
- $(\exists H)(\forall x)[U \geq x]$. (H is for Highest).
- We use H freely in this sentence. They have the meaning above.
 $(\forall x \neq H)(\exists y)x < y \wedge (\forall z)[\neg(x < z < y)]$.
(For every element $x \neq H$ there is a *successor*.)
- We use L freely in this sentence. They have the meaning above.
 $(\forall x \neq L)(\exists y)y < x \wedge (\forall z)[\neg(y < z < x)]$.
(For every element $x \neq L$ there is a *predecessor*.)

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4. (10 points) Let a_i be defined as follows:

$$a_0 = 40$$

$$a_1 = 17$$

$$a_2 = 40$$

$$(\forall n \geq 3)[a_n = 3a_{n-1} + 2a_{n-2} + a_{n-3} + 7].$$

Use constructive induction to find an b, m such that, for all n , $a_n \equiv b \pmod{m}$.

SOLUTION

IB: Need

$$40 \equiv b \pmod{m}.$$

$$17 \equiv b \pmod{m}.$$

$$40 \equiv b \pmod{m}.$$

IH: For all $n' < n$, $a_{n'} \equiv b \pmod{m}$.

IS: All \equiv are mod m .

$$a_n = 3a_{n-1} + 2a_{n-2} + a_{n-3} + 7 \equiv 3b + 2b + b + 7 = 6b + 7$$

So we need

$$6b + 7 \equiv b$$

$$5b + 7 \equiv 0.$$

Se we need

$$10 \equiv b \pmod{m}.$$

$$17 \equiv b \pmod{m}.$$

$$40 \equiv b \pmod{m}.$$

$$5b + 7 \equiv 0.$$

Note that we need 17, 40 are all equivalent to each other mod m .

$$40 \equiv 17 \text{ so } 23 \equiv 0.$$

Lets try $m = 23$.

$$40 \equiv 17 \pmod{23}.$$

$$17 \equiv 17 \pmod{23}.$$

So it looks like $b = 17$.

Lets see if the last equation works:

$$5b + 7 \pmod{23} \equiv (5 \times 17) + 7 \pmod{23}.$$

To avoid big numbers I do this as

$$2 \times 17 + 2 \times 17 + 17 + 7 \equiv 34 + 34 + 17 + 7 \equiv 11 + 11 + 24 \equiv 22 + 24 \equiv -1 + 1 \equiv 0.$$

So YES, $b = 17$ and $m = 24$.

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5. (10 points) Use unique factorization to show the following:

Let $n \in \mathbb{N}$, and $n \geq 2$. Let $n = p_1^{n_1} \cdots p_k^{n_k}$, where p_1, \dots, p_k are primes. If $n^{1/7}$ is rational then $(\forall 1 \leq i \leq k)[n_i \equiv 0 \pmod{7}]$.

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6. (10 points) Bill makes his Darling lunch. Let $s, f, d \in \mathbb{N}$.

Her lunch will be a s different sandwich's, f different fruits, and d deserts some of which can be the same.

Example: If $s = 2$, $f = 3$, and $d = 4$ then Darling could have

SANDWICHES: An egg salad sandwich, a PBJ sandwich.

FRUIT: An apple, an orange, a grape.

DESERT: 2 Poptarts, 1 cookie, 1 Hershey kiss (from her Bill!).

There are S possible sandwiches ($s < S$).

There are F possible fruits ($f < F$).

There are D possible Deserts ($d < D$).

(a) (5 points) How many ways can Darling have lunch?

(b) (5 points)

(c) One of the sandwiches is Peanut butter and Blueberry jam, which we denote PBJ

One of the fruits is a BLUEBERRY.

One of the deserts is BLUEBERRY PIE.

Darling says *I am okay with having 2 of the 3 items be blueberry-based but I DO NOT want all three of them to be blueberry based.*

How many ways can Darling have lunch?

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7. (10 points) How many 5-tuples of natural numbers $(x_1, x_2, x_3, x_4, x_5)$ are such that

$$x_1 + x_2 + x_3 + x_4 + x_5 = 1000$$

and $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, $x_4 \geq 4$, and $x_5 \geq 5$.

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8. (10 points) Let $r, s \in \mathbb{N}$.

Martians play poker with cards that each have

a rank in $\{1, \dots, r\}$, and

a suite in $\{1, \dots, s\}$.

They play a 6-card hand.

- (a) What is the probability of getting two three-of-a-kind? We denote this $p_{3,3}(r, s)$.
- (b) What is the probability of getting four-of-a-kind? We denote this $p_4(r, s)$.
- (c) Does there exist an r, s such that $p_{3,3}(r, s) < p_4(r, s)$. If so then produces such an r, s . If not then say why not.
- (d) Does there exist an r, s such that $p_{3,3}(r, s) > p_4(r, s)$. If so then produces such an r, s . If not then say why not.