

- 1. (0 points) Write your name! READ cipher and english.
- 2. (20 points) Alice and Bob want to use a 1-time pad but don't want to exchange 10^{100} bits. So they do the following:
 - Alice picks a number *n* between 1 and 1000 at random and sends it to Bob.
 - They both compute 1/n in base 10 and get a sequence of digits. They then take each digit and write it in base 2. This gives them both a shared infinite sequence of bits. (NOTE that in the case of a number that has a finite representation, pad infinitely with 0's.)

Example: n = 7. 1/7 = 0.1428714287. So use the key (which I write with spaces for your understanding)

 $1 \ 100 \ 10 \ 1000 \ 111 \ 1 \ 100 \ 10 \ 1000 \ 111 \ \cdots$

Another Example: n = 10. $1/10 = .10000 \cdots$ so use the key

 $1 \ 0 \ 0 \ 0 \ 0 \ \cdots$

- (a) (5 points) Alice picks n = 13. She then wants to send the message 0110. What does she send? (Show all of your work.)
- (b) (5 points) Alice picks n = 10. She then wants to send the message 11001. What does she send?
- (c) (5 points) Find a number n such that 1/n is of the form $0.YXXXX \cdots$ where X is at least 10 digits long (Y can be any length or even empty).
- (d) (5 points) Discuss the PROS and CONS of Bob and Alice's modification to the 1-time pad.
- 3. (20 points) Describe carefully how Alice and Bob can do a **VIG PLUS PLUS CIPHER** (the two PLUS's in a row are intentional) where they use a 2 × 2 matrix cipher instead of affine or shift. Also describe how Eve can crack it.

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- 4. (20 points) Alice and Bob want to use a 1-time pad, but they want to use sequences of digits instead of sequences of bits. How can they do this?
- 5. (20 points) Alice wants to use a general 2-char cipher. Bob wants to use a 2×2 matrix cipher (henceforth just *matrix cipher*).
 - (a) Give reasons why the General 2-char cipher is better than the matrix cipher.
 - (b) Give reasons why the matrix cipher is better than the General 2-char cipher.

(NOTE- THE reason you can argue both is that I did not define 'better' carefully.)

- 6. (20 points)
 - (a) (7 points) Give an example of a 2×2 matrix over mod 26 that IS invertible.
 - (b) (7 points) Give the inverse of the matrix you just gave.
 - (c) (6 points) Give an example of a 2×2 matrix over mod 26 that IS NOT invertible. No explanation needed.