1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? Where is the midterm? When is the Final?

2. (60 points) RECALL the following formal statement of the Pumping Theorem:

If \( L \) is regular then there exists \( N \) such that, for all \( w \in L \), \(|w| \geq N\), there exist \( x, y, z \), \( y \neq e \), such that (1) \( w = xyz \) and (2) \((\forall i)[xy^iz \in L]\).

In this problem you will prove a variant of this. Prove the following:

If \( L \) is regular then there exists \( N_1 \) and \( N_2 \) such that, for all \( w \in L \), \(|w| \geq N_1 \), there exist \( x, y, z \), \( y \neq e \), such that (1) \( w = xyz \) and (2) \(|y| \leq N_2 \) and (3) \((\forall i)[xy^iz \in L]\).

3. (40 points) Let \( n \geq 2 \). Let \( A_1, A_2, \ldots, A_n \) be countable

   (a) Show that \( A_1 \cup \cdots \cup A_n \) is countable.

   (b) Show that \( A_1 \times \cdots \times A_n \) is countable.