HW 1 CMSC 452. Morally DUE Feb 7

NOTE- THIS HW IS THREE PAGES LONG!!!

THROUGOUT THIS HW YOU CAN ASSUME:

- The union of a finite number of countable sets is countable.

- The union of a countable number of finite sets is countable (not quite true if they are all the same set, but avoid that case).

- The union of a countable number of COUNTABLE sets if countable.

- The cross product of a finite number of countable sets is countable.

- The following sets are countable: N, Z, Q.

- The following sets are uncountable: $(0, 1)$, R.

1. (0 points) READ UP ON COUNTABILITY ON THE WEB. READ MY NOTES ON THE $HARD$ HIERARCHY- WHICH WILL BE AVAIL-ABLE LATER. What is your name? Write it clearly. Staple your HW. When is the midterm? Where is the midterm? When is the Final? IMPORTANT- I WANT TO MAKE SURE I HAVE YOUR CORRECT EMAIL ADDRESSES. I HAVE EMAILED ALL OF YOU USING WHAT I CURRENTLY THINK IS YOUR EMAIL ADDRESS BUT IF YOU DIDN'T GET IT THEN EMAIL ME ASAP TO GIVE ME YOUR REAL EMAIL ADDRESS.

2. (40 points) For each of the following sets say if its is

- Empty
- Finite but not empty
- Countable (this implies NOT finite)
- Uncountable

And EXPLAIN your answer.

NOTE: Throughout this HW $\mathsf{N} = \{1, 2, 3, \ldots\}$ it does NOT include 0.

(a) The set of all functions from N to $\{0, 1, 2\}$

(b) The set of all functions from N to $\{0, 1\}$

(c) The set of all functions from N to $\{0\}$

(d) The set of all functions from N to $\emptyset$

(e) The set of all functions from N to N that are INCREASING (so $x < y$ implies $f(x) \leq f(y)$).

(f) The set of all functions from N to N that are strictly INCREASING (so $x < y$ implies $f(x) < f(y)$).

(g) The set of all functions from N to N that are DECREASING (so $x < y$ implies $f(x) \geq f(y)$).

(h) The set of all functions from N to N that are strictly DECREASING (so $x < y$ implies $f(x) > f(y)$).

3. (30 points) Let the $BILL_i$ numbers be defined as follows:

- $BILL_0 = \mathbb{Q}$ (the rationals)
- $BILL_{i+1}$ is the union of the following three sets:
  - $BILL_i$
  - $\{x + y : x, y \in BILL_i\}$
  - $\{x^y : x, y \in BILL_i\}$.

Let $BILL = \bigcup_{i=0}^{\infty} BILL_i$.

(a) Is $BILL$ countable or uncountable? Proof your result.

(b) Let $BILL[x]$ be the set of polynomials with coefficients in $BILL$. Let $BILLBILL$ be the set of all roots of equations in $BILL[x]$. Is $BILLBILL$ countable or uncountable? Proof your result.

4. (30 points) Show that $7^{1/3}$ does not satisfy any quadratic equation over the integers using the method shown in class.