HW 2 CMSC 452. Morally DUE Feb 14
THIS HOMEWORK IS THREE PAGES

1. (0 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW.

2. (10 points) (RECALL that $\#_a(w)$ is the number of a's in w, $\#_b(w)$ is ....) Show that the following set is regular by drawing a DFA for it

$$\{w : \#_a(w) \equiv 0 \pmod 2 \vee \#_b(w) \equiv 1 \pmod 3\}$$

3. (20 points) Let $n_1, n_2 \geq 3$.

   (a) Consider

   $$L_1 = \{w : \#_a(w) \equiv 0 \pmod{n_1} \vee \#_b(w) \equiv 1 \pmod{n_2}\}$$

   How many states is in the smallest DFA for this L? How many of those states are accepting states?

   (Some books use 'final states' for 'accepting states'.)

   (b) Consider

   $$L_2 = \{w : \#_a(w) \equiv 0 \pmod{n_1} \wedge \#_b(w) \equiv 1 \pmod{n_2}\}$$

   How many states is in the smallest DFA for this L? How many of those states are accepting states?

   (Some books use Final states for Accepting states.)

   ### THERE ARE TWO MORE PAGES

4. (30 points) We interpret strings over $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as numbers in base 10. A DFA CLASSIFIER is a DFA where instead of having final states has each state is labeled, so we think of the DFA as computing a function. (If on string w you end up at state q then we think of w as being mapped to the label of q.)

(a) IF you were to write a DFA CLASSIFIER that will, on input w, tell what w is congruent to mod 11 THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)

(b) IF you were to write a DFA CLASSIFIER that will, on input w, tell what w is congruent to mod 13 THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)

(c) Describe a procedure that does the following: Given n, find the size of a DFA classifier that will, on input w, tell what w is congruent to mod n.

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5. (40 points)

   (a) (0 points) Draw a NDFA for the set

   $$\{(A, B) \mid (\exists x)[x \in A \land x \in B]\}$$

   (YES this is the one I did in class, but wait for the next few.)
   How many states does it have?

   (b) (0 points) Draw a NDFA for the set

   $$\{(A, B) \mid (\exists x)[x + 1 \in A \land x \in B]\}$$

   (YES this is the one I did in class, but wait for the next few.)
   How many states does it have?

   (c) (0 points)
   Draw a NDFA for the set

   $$\{(A, B) \mid (\exists x)[x + 2 \in A \land x \in B]\}$$

   (YES this is the one I did in class, but wait for the next few.)
   How many states does it have?

   (d) (10 points)
   Draw a NDFA for the set

   $$\{(A, B) \mid (\exists x)[x + 5 \in A \land x \in B]\}$$

   (YES this is the one I did in class, but wait for the next few.)
   How many states does it have?

   (e) (30 points) Fix $k \in \mathsf{N}$. Draw a NDFA for the set $\{(A, B) \mid (\exists x)[x + k \in A \land x \in B]\}$ You may use ... notation and will have to; however, make your use of ... (and your answer in general) so clear that anyone looking at your answer will be able to, given k, How many states does it have?