HW 4 CMSC 452. Morally DUE Feb 28
THIS HOMEWORK IS TWO PAGES
SOLUTIONS

1. (0 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)

2. (30 points)

   (a) Write a DFA for $\{(X, x) : x \in X \wedge x \equiv 0 \pmod 3\}$

   (b) Write an NDFA for $\{X : (\exists x)[x \in X \wedge x \equiv 0 \pmod 3]\}$

3. (30 points)

   (a) Write a DFA for: $\{(X, x) : x \in X \wedge x \equiv 0 \pmod 4\}$.

   (b) Let $k \geq 5$. Write a DFA for:
   $\{(X, x) : x \in X \wedge x \equiv 0 \pmod k\}$.
   (You may use ... notation- you'll probably have to.)

4. (40 points) For this problem we consider a regular expression to use the symbols:

   $\{ \quad a \quad b \quad \cup \quad e \quad \} \quad \cdot \quad *$

   Each of these symbols has length 1.

   (a) Give equations for (1) $R(i, j, 0)$ and (2) $R(i, j, k)$ (based on $R(*, *, k-1)$).

   (b) What is the longest that $R(i, j, 0)$ can be? (We allow either $i = j$ or $i \neq j$ whichever one gives the longest $R(i, j, 0)$.)

   (c) Assume that $(\forall i, j)[|R(i, j, k-1)| \leq L$. Give a bound L' such that $(\forall i, j)[|R(i, j, k)| \leq L'$.

   (d) Use the answer for part b to find a function $f(k)$ that, for all i, j, k, $|R(i, j, k)| \leq O(f(k))$.

   (e) Fill in the following sentence: If a regular language has a DFA with n states then it has a Reg Exp of length $\leq O(XXX)$.

SOLUTION TO PROBLEM 4

a) Omitted

b) If $i = j$ then $R(i, j, 0)$ could be $\{a, b\} \cup \{e\}$ which is of length 9.

c)

$R(i, j, k) = R(i, j, k-1) \cup R(i, k, k-1) \cdot R(k, k, k-1)^* \cdot R(k, j, k-1)$

Note that $\cup$ and both $\cdot$'s and the $*$ are 4 symbols. So

$$|R(i, j, k)| = 4+|R(i, j, k-1)|+|R(i, k, k-1)|+|R(k, k, k-1)|+|R(k, j, k-1)| \leq 4+4L$$

SO $L' = 4 + 4L$.

d) From b and c we have

$f(0) = 9$

$f(k) = 4 + 4(f(k-1)$

From this one can show that $f(k) = O(4^k)$.

e) If a DFA has n states then the regular expression for it will be a union of reg exps of the form $R(1, f, n)$ where f is a final state. There could be LOTS of final states- at most n (though if there are n then you can use a muchs smaller DFA since the lang is Σ^*). Hence the reg expression is of size $\leq O(n4^n)$.