**HW 5 CMSC 452. Morally DUE Mar 7**
**SOLUTIONS**

1. (5 points) What is your name? Write it clearly. When is the midterm? Write that clearly too. Staple your HW. WHAT IS THE DAY/TIME OF THE MIDTERM? (HINT: The Midterm is March 30 IN CLASS at 11:00.)

2. (40 points)

   (a) (Use our usual convention for pairs of numbers, so the alphabet is $\{00, 01, 10, 11\}$, which we usually write vertically.)

   Write a DFA for $\{(x, y) : x = y + 3\}$. Label each state

   - A for accept,
   - R for reject, or
   - B for Bad Format.

   (NOTE - the comma after "reject" is called an *Oxford comma.*)

   (b) Write a DFA for $\{(x, y) : x \neq y + 3\}$. Label each state

   - A for accept,
   - R for reject, or
   - B for Bad Format.

   SOLUTION FOR PROBLEM 2.

   Mostly omitted. I will note that the second problem can be solved by taking the DFA from the first problem and swapping the ACCEPT and REJECT, but leaving the BAD FORMAT states as they are.

3. (25 points) $A \subseteq \{0,1\}^\omega$ is $D$-regular if there is a DFA $M = (Q, \Sigma, \delta, s, F)$ such that:

   $x \in A \rightarrow$ if you run $M$ on $x$ you hit a state in $F$ infinitely often

   $x \notin A \rightarrow$ if you run $M$ on $x$ you DO NOT hit a state in $F$ infinitely often

   (a) Write a $D$-DFA for

   $$L_1 = \{X : X \text{ is infinite }\}.$$

   (b) Take the $D$-DFA you wrote for $L_1$ in part a. Swap the final and non-final states. Let $L_1'$ be the language (subset of $\{0,1\}^\omega$) that your new automta accepts. Describe $L_1'$. Is it $\overline{L_1}$ (also called the *compliment* of $L_1$ or $\{0,1\}^\omega - L_1$)?

   (c) Write a $D$-DFA for

   $$\{X : X \text{ is infinite and } \mathsf{N} - X \text{ is infinite }\}.$$

   (d) Write an $D$-DFA for

   $$\{X : \text{ there is an infinite number of } x \text{ such that } x \in X \text{ and } x + 1 \in X\}$$

   (e) Show this is NO $D$-DFA for $\{X : X \text{ is finite}\}$.

**SOLUTION TO PROBLEM 3**

3.1) This one is so easy I'll give it in table form

$Q = \{0,1\}$ (0 means that you have not seen a 1, 1 means that you just have.)

$\Sigma = \{0,1\}$

$\delta(0,0) = 0$

$\delta(0,1) = 1$

$\delta(1,0) = 0$

$\delta(1,1) = 1.$

Upshot: whenever a 1 comes in you go to (or stay in) state 1.

$F = \{1\}$

3.2)

$Q = \{0, 1\}$ (0 means that you have not seen a 1, 1 means that you just have.)

$\Sigma = \{0, 1\}$

$\delta(0, 0) = 0$

$\delta(0, 1) = 1$

$\delta(1, 0) = 0$

$\delta(1, 1) = 1$.

Upshot: whenever a 1 comes in you go to (or stay in) state 1.

$F = \{0\}$

$L_1'$ is the set of sets $A$ where $\mathsf{N} - A$ is infinite.

The complement of $L_1$ is $\{X : X \text{ is finite }\}$. This is NOT the complement of $L_1$. for example

$L_1'$ has $\{0, 2, 4, 6, \ldots\}$.

The complement of $L_1$ does not have that.

3.3) Omitted.

3.4) Omitted.

3.5) Assume there is a DFA for $\{X : X \text{ is finite }\}$.

Feed in 0's until you get to a FINAL state. This must happen since if the rest are all 0's you would hit a final state infinitely often.

Then feed in a 1. Then feed in 0's until you get to a final state. This must happen since if the rest are all 0's you would hit a final state infinitely often.

Then feed in a 1. Then feed in 0's until you get to a final state. This must happen since if the rest are all 0's you would hit a final state infinitely often.

Keep doing this. You will produce an infinite set that is accepted.

4. (35 points) A *J-automata* $M$ is a tuple $(Q, \Sigma, \delta, s, F)$ such that

- $Q$ is a set of states - just like in a DFA.
- $\Sigma$ is an alphabet - just like in a DFA.
- $\delta : Q \times \Sigma \to Q$ - just like a DFA.
- $s \in S$, the start state - just like a DFA.
- $F$ is a NOT a subset of $Q$. $F$ is a set of subsets of $F$. For example, if $Q = \{1, 2, 3, 4, 5, 6\}$ $F$ could be $\{\{1, 2, 5\}, \{1, 5\}, \{2, 3, 6\}\}$

Let $x \in \{0, 1\}^\omega$. We say that $J$-automta $A$ *accepts* $x$ if, when you run $x$ through $A$, the set of states that are visiting infinitely often is a set in $F$. For example, in the above example, if the set of states visiting infinitely often was $\{1, 2, 5\}$ then ACCEPT, but if its $\{1, 2\}$ then REJECT.

A subset of $\{0, 1\}^\omega$ which is accepted by an $J$-automta is called $J$-regular.

(a) (0 points) How would you compliment an $J$-regular set?

(b) (7 points) Show that if $L$ is $J$-regular then $\overline{L}$ is $J$-regular.

(c) (7 points) Give a $J$-automata for $\{X : X \text{ is infinite } \}$.

(d) (7 points) Give a $J$-automata for $\{X : X \text{ is finite } \}$.

(e) (7 points) Give a $J$-automata for $\{X : X \text{ is infinite and } \mathsf{N} - X \text{ is infinite}\}$.

(f) (7 points) Let $T = \{x : x \equiv 0 \pmod 3\}$. Give a $J$-automata for $\{X : X \cap T \text{ is infinite}\}$.

**SOLUTION TO PROBLEM 4**

4.1) There are many answers. Here is an example:

*Oh, like the way you accept infinite strings!*

4.2) If $L$ is recognized by $(Q, \Sigma, \delta, s, F)$ then

$\overline{L}$ is recognized by $(Q, \Sigma, \delta, s, 2^Q - F)$.

4.3) Same DFA as in solution to Problem 3.1 except that the the final states are now the SET of states $\{\{0, 1\}, \{1\}\}$.

4.4) Same DFA as in solution to Problem 3.1 except that the the final states are now the SET of states $\{\{0\}\}$.

4.5) Same DFA as in solution to Problem 3.1 except that the the final states are now the SET of states $\{\{0, 1\}\}$.

4.5) Omitted.