HW 11 CMSC 452. Morally Due May 2
THIS HW IS TWO PAGES!

1. (0 points BUT if you don't do it you'll get a 0 on the entire HW) What is your name? Write it clearly. Staple the HW.

2. (20 points)

   Format (the student): Turing Machines should have the freedom to move L or R AND print!

   Bill (the professor): Rather than argue the point I'll make a problem set out of it!

   (a) DEFINE a Turing machine that can, on seeing an input and of course knowing its state, will change its state (or not) and: either (1) Move L and print a character, (2) Move R and print a character, (3) Stay at the same position and print a character. (You need only tell me what the δ function does- what is its domain and range.)

   (b) In the proof of Cook's theorem we needed to, for every type of instruction, have a formula. With these NEW Turing Machines do the following: For the instruction that on input a, in state p goes to state q, moves L and prints a b, give the formula that captures that. Do not worry about The formula will use variables of the form $x_{i,j,}$ with the third part being either an element of Σ or of $\Sigma \times Q$.

3. (20 points) 3) Let

$$COL = \{(G, k) : G \text{ is } k\text{-colorable } \}$$

Let

$$FCOL(G) = \text{ [the least } k \text{ such that } G \text{ is } k\text{-colorable]}$$

Show that if $COL \in P$ then $FCOL \in FP$ (functions computable in poly time).

How many queries to COL did your algorithm make?

4. (30 points) Let

$$COL = \{(G, k) : G \text{ is } k\text{-colorable } \}$$

(as before)

Let $FCOL2(G)$ be an actual optimal coloring. That is, the output is G together with a coloring of G with $FCOL(G)$ colors.

Show that if $COL \in P$ then $FCOL2 \in FP$.

How many queries to COL did your algorithm make?

5. (30 points) Consider the following statement:
There is a way to compute $FCOL2$ with $O(\log n)$ queries to COL.

Which of the following is true? Prove your result.

- KNOWN: the statement is TRUE.
- KNOWN: Assuming $P \neq NP$ the statement is TRUE.
- KNOWN: the statement is FALSE.
- KNOWN: Assuming $P \neq NP$ the statement is FALSE