452 FINAL- VERSION E

Do not open this exam until you are told.

Read these instructions:

1. This is a closed book exam, though *one* sheet of notes is allowed. **No calculators, or other aids are allowed**. If you have a question during the exam, please raise your hand.
2. There are 5 problems which add up to 100 points. The exam is 2 hours.
3. For each question show all of your work and **write legibly**. **Clearly indicate** your answers. No credit for illegible answers.
4. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
5. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

6. Fill in the following:

NAME :
SIGNATURE :
SID :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
Prob 5:	
TOTAL	

1. (20 points, 5 points each) Give an example of each of the following. **No proofs are needed**; however, make sure your answer is clear.

   (a) Give a regular language L such that (1) any DFA for L requires at least 10,000 states, but (2) there is an NFA for L with ≤ 1000 states.

   SOLUTION:

   $$\{a^i : i \neq 10,000\}$$

   END OF SOLUTION:

   (b) Give a set that is NP-complete, where the input is **not** a set of Boolean formulas. (NOTE- I want a SET not a FUNCTION.)

   SOLUTION:

   $$\{G : G \text{ is 3-colorable } \}$$

   Can also use CLIQUE, IND SET, others that were not in class but people happened to know about were fine.

   END OF SOLUTION:

   (c) Give a set that is in NP but (1) the question of whether or not it is in NP-complete is **unknown to science**, and (2) the question of whether or not it is in P is **unknown to science** (NOTE- I want a SET not a FUNCTION.)

   SOLUTION:

   $$\{(n, m) : \text{ this is a factor of } n \text{ that is } \le m \ \}$$

   NOTE- I wanted a SET, not a FUNCTION or a WORD. If you just said FACTORING that was worth 2 out of 5 points.

   Another answer is

   $$\{(G, H) : \ G \text{ and } H \text{ are isomorphic } \}$$

   END OF SOLUTIONS

(d) Give three examples of undecidable SETS.

SOLUTION:

I'll give MANY examples

$$\{e : M_e(e) \text{ halts } \}$$

$$\{e : M_e(e) \text{ halts } \} - F$$

(where F is some finite set)

$$\{e : M_e(e) \text{ halts } \} \cup F$$

(where F is some finite set)

$$\{e : M_e \text{ halts on all primes}\}$$

$\{e : M_e$ does the hokey pokey and turns itself about, cause thats what its all about! $\}$

$$\{p(x_1, \ldots, x_{14}) : (\exists a_1, \ldots, a_{14} \in Z)[p(a_1, \ldots, a_{14}) = 0]\}$$

And one student happened to know the following:

The set of all tuples of matrices $(M_1, \ldots, M_n)$ over the integers such that some product of them (allowed to repeat them) is the 0-matrix.

END OF SOLUTION

2. (20 points) We use the WS1S convention. Recall that, for example $(2,5)$ is represented as follows:

$$\begin{array}{ccccccc} 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * \end{array}$$

where the $*$ can be either 0 or 1.

Write down DFA's for the following. Label states A (for Accept), R (for Reject), and S (for Stupid).

(a) $\{(x,y) : x \equiv y \pmod{5}\}$.

(b) $\{(x,y) : x \equiv y \pmod{1000}\}$. (For this one, you can and should use "$\ldots$")

(The next page is blank. Do the mod-5 problem on this page and the mod-1000 problem on the next page.)

SOLUTION OMITTED

3. (20 points) For each of the following say if its TRUE, FALSE, or UNKNOWN TO SCIENCE. No Proof Required BUT you get $+4$ for every right answer and -3 for every wrong answer and 0 for an answer left blank. So

   DO NOT GUESS!!!!!!!!!!!!!!!!!!!!!!!!!!!

   (a) If A is regular and F is a finite set then $A \cup F$ is regular.

   (b) If A is in P and F is a finite set then $A \cup F$ is in P.

   (c) If A is in NP and F is a finite set then $A \cup F$ is in NP.

   (d) If A is decidable and F is a finite set then $A \cup F$ is decidable.

   (e) If A is undecidable and F is a finite set then $A \cup F$ is undecidable.

   SOLUTION

   They are ALL TRUE! When I said DO NOT GUESS this is why- some students told me that they knew all but two of them and guessed they were both FALSE since *Dr. Gasarch wouldn't make them all TRUE.* Yes he would!

   For the first four you can hardcode in that finite set. The last one I leave to you.

   END OF SOLUTION

4. (20 points) Let

$$CLIQ_\alpha = \{G : G \text{ has a clique of size } \geq \alpha n\}$$

where n is the number of vertices in G.

Show that

$$CLIQ_{1/3} \leq CLIQ_{2/3}$$

(You must give the reduction; you can't just say they are both NP-complete, though they are.)

SOLUTION

We need to show how to, given a graph G on n vertices create a graph G' on n' vertices such that

G has a clique of size $n/3$ iff G' has a clique of size $2n'/3$

G' is G with n vertices added that are connected to each other and to all elements of G.

note that $n' = 2n$.

If G has a clique of size $n/3$ then G' has a clique of size $(n/3) + n = 4n/3 = (2/3)n'$.

If G' has a clique of size $2n'/3 = 4n/3$ then G has a clique of size $n/3$.

You NEED to do a reduction

You CANNOT assume you have a procedure to find clique

You CANNOT remove vertices- that does not work.

END OF SOLUTION

5. (20 points) Prove that there exists a decidable set that is **not** in NP.

SOLUTION

If A is in NP then there exists poly p and set $B \in P$ such that

$$A = \{x : (\exists y)[|y| = p(|x|) \wedge (x, y) \in B\}$$

By going through ALL possibly Y, $A \in DTIME(2^{p(n)})$. Hence

$$NP \subseteq \bigcup_{i=1}^{\infty} DTIME(2^{n^i})$$

Now do the standard proof that there is a decidable set that is NOT $\bigcup_{i=1}^{\infty} DTIME(2^{n^i})$.

Some students tried to list out all NP machines and diag against them. This is very hard to get right. You would need to do the following kind of thing

List out all TM's $M_1, M_2, \ldots$

Clock them via M_i clocked with $n^i + i$.

Adjust them to take two inputs x, y.

List out all polynomials $p_1, p_2, \ldots$.

NP can be represented by M_i, p_j where M_i is the verified and p_j is the length of the witness.

(a) Input $a^i b^j$

(b) Run $M_i(a^i b^j, y)$ for all y with $|y| = p_j(|i + j|)$.

(c) If EVERY answer is NO then output yes.

(d) If SOME answer is YES then output no.

Scratch Paper