

HW 2 CMSC 452. Morally DUE Feb 13
THIS HOMEWORK IS TWO PAGES

1. (0 points) What is your name? Write it clearly. Staple your HW.
2. (25 points) Show that the following set is regular by drawing a DFA for it

$$\{w : n_a(w) \equiv 1 \pmod{2} \wedge n_b(w) \equiv 1 \pmod{5}\}$$

3. (25 points) Let $n_1, n_2 \geq 5$. Let $0 \leq a_1 \leq n_1 - 1$. Let $0 \leq a_2 \leq n_2 - 1$.
 - (a) Consider

$$L = \{w : n_a(w) \equiv a_1 \pmod{n_1} \wedge n_b(w) \equiv a_2 \pmod{n_2}\}$$

Describe a DFA that accepts L . How many states does it have? How many accept states does it have?

(Some books use Final states for Accepting states.)

- (b) If you did the last problem correctly the number of states was $n_1 n_2$. TRUE or FALSE: There exists a_1, a_2, n_1, n_2 with $n_1, n_2 \geq 5$ and $0 \leq a_1 \leq n_1 - 1$, $0 \leq a_2 \leq n_2 - 1$ such that there is a DFA for L with MUCH LESS than $n_1 n_2$ states. Justify your answer. If TRUE then show such an a_1, a_2, n_1, n_2 and the small DFA for L . If FALSE then just say FALSE- no proof needed.

THERE IS ONE MORE PAGE

4. (25 points)
- (a) We interpret strings over $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ as numbers in base 10. A DFA CLASSIFIER is a DFA where instead of having final states has each state is labeled, so we think of the DFA as computing a function. (If on string w you end up at state q then we think of w as being mapped to the label of q .) IF you were to write a DFA CLASSIFIER that will, on input A BASE 10 NUMBER w , tell what w is congruent to mod 15, THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)
 - (b) We interpret strings over $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7\}$ as numbers in base 8. IF you were to write a DFA CLASSIFIER that will, on input A BASE 8 NUMBER w , tell what w is congruent to mod 6 THEN how many states would it have. Explain. (You need not write the actual DFA classifier.)
 - (c) Describe a procedure that does the following: Given b, n finds the size of a DFA classifier that will, on input A BASE b number w , tell what w is congruent to mod n .
5. (25 points) Show that if L is regular than L^* is regular. (Take an DFA for L and use it to create an NDFA for L^* . Note that the empty string is in L^* .)