HW 3 CMSC 452. Morally DUE Feb 20
THIS HOMEWORK IS THREE PAGES

USE our convention for representing:

- a number (n = [0 0 0 0 . . . 0 1 * * * . . .], where there are n 0s)
- a tuple of numbers
- a tuple of numbers and finite sets

1. (0 points) What is your name? Write it clearly. What day is the midterm? Staple your HW.
2. (40 points) For each of the following languages (lettered a-d), draw a DFA. Make sure it has ACCEPT states, REJECT states, and STUPID states. Then, answer each of these questions about the DFA you drew:

   (i) How many ACCEPT states does it have?

   (ii) How many REJECT states does it have?

   (iii) How many STUPID states does it have?

   (a) $\{(x, y) : x = y + 2\}$.

   (b) $\{(x, y) : x \neq y + 2\}$. (Note the $\neq$ here!)

   (c) $\{(x, y) : x = y + 100\}$. (For this one, you can and should use DOT DOT DOT rather than have LOTS of states.)

   (d) $\{(x, y) : x \neq y + 100\}$. (For this one, you can and should use DOT DOT DOT rather than have LOTS of states.)

THERE ARE TWO MORE PAGES

3. (30 points) Consider the sentence

$$(\exists X)(\forall x)(\exists y)[(x \in X) \wedge (y \notin X) \wedge (x = y + 100)\}]$$

I would want to ask you to build the DFA's needed to decide if this sentence is true or false. That would be madness! Instead, I'll ask you about parts of the process and about number-of-states.

(a) Draw a DFA for $\{(x, X) : x \in X\}$ How many states does it have?

(b) Draw a DFA for $\{(y, X) : y \notin X\}$ How many states does it have?

For the rest of this question, we will assume that

$$\{(x, y) : x = y + 100\}$$

can be done with 100 states (OK, it's really more like 104, but what's 4 states among friends?).

(c) Consider the language

$$\{(x, y, X) : [(x \in X) \wedge (y \notin X) \wedge (x = y + 100)\}]$$

DO NOT DRAW THE DFA! For that way lies madness!

But: how many states would the DFA for this have if you were to draw it? DO NOT be clever! Just take the answers to the prior 3 problems and use them. (You may be able to do better by looking at this particular problem, but I am trying to make a more general point.)

(d) Consider the language

$$\{(x, y)\ st(\exists X)[(x \in X) \wedge (y \notin X) \wedge (x = y + 100)\}]$$

Give an upper bound on how many states a DFA for this has, based on the prior problem. (You may be able to do better by looking at this particular problem, but I am trying to make a more general point. Use the NFA to DFA construction.)

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4. (30 points) A *Sekora DFA* is a DFA which we intend to run on infinite strings. We DEFINE a Sekora DFA M to ACCEPT $x \in \{0,1\}^\omega$ if, when you run x through the M and get an infinite sequence of states, an infinite number of them are FINAL states.

   Give an algorithm that will, given a Sekora DFA, determine if there exists ANY infinite string that it accepts.

   Hint: You may find it helpful to think of a DFA as a (finite) directed graph - think about what has to happen for us to be able to visit the same vertex repeatedly in the same path.

   (NOTE - we DO NOT ever actually run a DFA on an infinite string.)