HW 5 CMSC 452. Morally DUE Mar 6
THIS HOMEWORK IS TWO PAGES

1. (0 points) What is your name? Write it clearly. What day is the midterm? Staple your HW.

2. In this problem the alphabet is $\{a, b\}$.

   (a) (20 points) Write a regular expression for

   $$\{w : n_a(w) \equiv 0 \pmod 3\}.$$

   (b) (20 points) Write a regular expression for

   $$\{w : n_a(w) \equiv 1 \pmod 3\}.$$

3. Alphabet is $\{a\}$.

   (a) (10 points) Write a DFA for the language $\{a^i : i \neq 1000\}$ (you may use ...) How many states does it have?

   (b) (10 points) Let $n \in N$. Think of n as large. Write a DFA for the language $\{a^i : i \neq n\}$ (you may use ...) How many states does it have (this will be a function of n).

   (c) (0 points but please think about – Please do so by the REAL day its due, March 6 so we can discuss in class) The answer to the last part was roughly n (for example, it might be $n + 1$). Is the following true or false: *Any NFA for L requires around n states.* Try to prove or disprove.

TURN THE PAGE

4. In this problem we go through a VERY simple case of going from a DFA to a regular expressio. DO NOT CHEAT- follow the construction. The alphabet is $\{a, b\}$.

   (a) (10 points) Write an NFA for the language

   $$L = \{w : \ a \text{ is the first letter of } w \ \}$$

   that has only two states. Label the two states 1 and 2 where 1 is the start state and 2 is the other state (which is the only final state).

   (b) (27 points) Compute, in order, and using the algorithm show in class. Show all steps.

   $R(1, 1, 0)$
   $R(1, 2, 0)$
   $R(2, 1, 0)$
   $R(2, 2, 0)$

   $R(1, 1, 1)$
   $R(1, 2, 1)$
   $R(2, 1, 1)$
   $R(2, 2, 1)$

   $R(1, 2, 2)$ (this is the only one I need)

   SHORT CUTS YOU CAN USE (You can use other ones also but be careful)

   For any reg exp α, $\emptyset \cdot \alpha = \emptyset$.

   If $\sigma \in \{a, b\}$ then $(\sigma \cup e)^* = \sigma^*$.

   $e^* = e$

   For any regular expression α, $e\alpha = \alpha$ and $\alpha e = \alpha$.

   For any $\sigma \in \Sigma$, $a \cup a = a$.

   (c) (3 points) From your work on part 1 write down a regular expression for L. (NOTE- it should be longer than the obvious reg exp for L which is $a(a \cup b)^*$.)