HW 5 CMSC 452. Morally DUE Mar 6
THIS HOMEWORK IS TWO PAGES
SOLUTIONS

1. (0 points) What is your name? Write it clearly. What day is the midterm? Staple your HW.

2. In this problem the alphabet is $\{a, b\}$.

   (a) (20 points) Write a regular expression for

   $$\{w : n_a(w) \equiv 0 \pmod 3\}.$$

   (b) (20 points) Write a regular expression for

   $$\{w : n_a(w) \equiv 1 \pmod 3\}.$$

   SOLUTION TO PROBLEM 2

   a) $b^*(b^*ab^*ab^*ab^*)b^*$.

   b) $b^*ab^*(b^*ab^*ab^*ab^*)b^*$.

   END OF SOLUTION TO PROBLEM 2

3. Alphabet is $\{a\}$.

   (a) (10 points) Write a DFA for the language $\{a^i : i \neq 1000\}$ (you may use $\ldots$) How many states does it have?

   (b) (10 points) Let $n \in N$. Think of n as large. Write a DFA for the language $\{a^i : i \neq n\}$ (you may use $\ldots$) How many states does it have (this will be a function of n).

   (c) (0 points but please think about – Please do so by the REAL day its due, March 6 so we can discuss in class) The answer to the last part was roughly n (for example, it might be $n + 1$). Is the following true or false: *Any NFA for L requires around n states.* Try to prove or disprove.

   SOLUTION TO PROBLEM 3

   Omitted

   END OF SOLUTION TO PROBLEM 3

TURN THE PAGE

4. In this problem we go through a VERY simple case of going from a DFA to a regular expressio. DO NOT CHEAT- follow the construction. The alphabet is $\{a, b\}$.

   (a) (10 points) Write an NFA for the language

$$L = \{w : \ a \text{ is the first letter of } w \ \}$$

   that has only two states. Label the two states 1 and 2 where 1 is the start state and 2 is the other state (which is the only final state).

   (b) (27 points) Compute, in order, and using the algorithm show in class. Show all steps.

   $R(1, 1, 0)$
   $R(1, 2, 0)$
   $R(2, 1, 0)$
   $R(2, 2, 0)$

   $R(1, 1, 1)$
   $R(1, 2, 1)$
   $R(2, 1, 1)$
   $R(2, 2, 1)$

   $R(1, 2, 2)$ (this is the only one I need)
   SHORT CUTS YOU CAN USE (You can use other ones also but be careful)
   For any reg exp α, $\emptyset \cdot \alpha = \emptyset$.
   If $\sigma \in \{a, b\}$ then $(\sigma \cup e)^* = \sigma^*$.
   $e^* = e$
   For any regular expression α, $e\alpha = \alpha$ and $\alpha e = \alpha$.
   For any $\sigma \in \Sigma$, $a \cup a = a$.

   (c) (3 points) From your work on part 1 write down a regular expression for L. (NOTE- it should be longer than the obvious reg exp for L which is $a(a \cup b)^*$.)

SOLUTION TO PROBLEM 4

a)

$\delta(1, a) = 2$

$\delta(1, b) = \emptyset$

$\delta(2, a) = 2$

$\delta(2, b) = 2.$

b)

$R(1, 1, 0) = e$

$R(1, 2, 0) = a$

$R(2, 1, 0) = \emptyset$

$R(2, 2, 0) = b \cup e$

$R(1, 1, 1) = R(1, 1, 0) \cup R(1, 1, 0)R(1, 1, 0)^*R(1, 1, 0) = e \cup ee^*e = e$

$R(1, 2, 1) = R(1, 2, 0) \cup R(1, 1, 0)R(1, 1, 0)^*R(1, 2, 0) = a \cup ee^*a = a$

$R(2, 1, 1) = R(2, 1, 0) \cup R(2, 1, 0)(R(1, 1, 0)^*R(1, 1, 0) = \emptyset \cup \emptyset \cdot = \emptyset.$

$R(2, 2, 1) = R(2, 2, 0) \cup R(2, 1, 0)R(1, 1, 0)^*R(1, 2, 0) = (b \cup e) \cup \emptyset \cdot = b \cup e$

$R(1, 2, 2) = R(1, 2, 1) \cup R(1, 2, 1)R(2, 2, 1)^*R(2, 2, 1) = a \cup a(b \cup e)^*(b \cup e) = ab^*b^*$

END OF SOLUTION TO PROBLEM 4