HW 6, CMSC 452. Morally DUE Mar 27

This HW is 200 points and counts twice as much as other HW

This HW is THREE PAGES LONG

(To make the midterm SHORTER, and to give you a break, there was NOT a question on countability. So, this is an additional HW just on that.)

QUESTION ONE (15 points each for 75 points total)

For each of the following sets, say if the set is:

FINITE (note: the empty set is countable)
COUNTABLE (that is, there is a bijection to N)
UNCOUNTABLE

(Note that a function must map **every** element of its domain.)
AND PROVE YOUR ANSWER.

1. The set of INCREASING functions from N to SQUARES. (A function f is INCREASING if $x < y$ implies $f(x) < f(y)$.)
2. The set of INCREASING functions from SQUARES to N. (A function f is INCREASING if $x < y$ implies $f(x) < f(y)$.)
3. The set of DECREASING functions from N to N (A function f is DECREASING if $x < y$ implies $f(x) > f(y)$.)
4. The set of DECREASING functions from N to Z (A function f is DECREASING if $x < y$ implies $f(x) > f(y)$.)
5. (For this homework, a function f from N to N is *kruskalian* if $x < y$ implies $f(x) \geq f(y)$ (NOTE $\geq$ NOT $>$).) The set of kruskalian functions from N to N.

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QUESTION TWO (65 points)

Consider the following proof that the rationals between 0 and 1 are uncountable. What is WRONG with the proof? (We ignore things like $.999\cdots = 1$, that is NOT the issue.)

Assume, by way of contradiction, that $Q \cap [0,1]$ is countable.

$$q_1, q_2, q_3, \ldots$$

be a listing of those rationals. We write them out with all of their digits:

$q_1 = .q_{11}q_{12}q_{13}\cdots$
$q_2 = .q_{21}q_{22}q_{23}\cdots$
$q_3 = .q_{31}q_{32}q_{33}\cdots$
$\vdots$

We will create a rational between 0 and 1 that is NOT on the list.

Let a hat ˆ over a number mean you add a 1 mod 10. so:

$\hat{0} = 1$
$\hat{8} = 9$
$\hat{9} = 0$.

The important thing is that $\hat{b} \neq b$.

We form the rational:

$$\hat{q_{11}}\hat{q_{22}}\hat{q_{33}}\cdots$$

This rational is NOT the ith on the list since it differs from q_i on the ith digit.

So the rationals between 0 and 1 are not countable.

WHAT IS WRONG WITH THIS PROOF?

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QUESTION THREE (60 points)

In your own words and pictures describe an algorithm that will:

Given a regular expression α, return an NFA that ACCEPTS exactly the strings that α GENERATES.