HW 9 CMSC 452. Morally Due April 10
SOLUTIONS

1. (0 points but you MUST READ THIS) For this HW $A \in DTIME(T(n)$ means that there is a JAVA program M such that

   If $x \in A$ then $M(x) \downarrow$ and outputs YES.

   If $x \notin A$ then $M(x) \downarrow$ and outputs NO.

   On input of length n M takes $O(T(n))$ steps.

2. (5 points) What is your name? Write it clearly. Staple the HW.

3. (20 points) Assume $L_1 \in DTIME(T_1(n))$ and $L_2 \in DTIME(T_2(n))$. Show that $L_1 \cap L_2 \in DTIME(T_1(n) + T_2(n))$. (You can write pseudo code and note how long the program runs. We ignore multiplicative and additive constants.)

   SOLUTION TO PROBLEM TWO

   $L_1 \in DTIME(T_1(n))$ via M_1, $L_2 \in DTIME(T_2(n))$ via M_2.

   (a) Input(x) of length n.
   (b) Run $M_1(x)$. If it returns NO then output NO and halt.
   (c) (If you got here then $M_1(x)$ returned YES). Run $M_2(x)$. If it returns NO then output NO and halt. If it returns YES then output YES and halt.

   The major time spend was in running $M_1(x)$, which takes $T_1(n)$ steps, and by running $M_2(x)$, which takes $T_2(n)$ steps. Hence the total time is

   $T_1(n) + T_2(n)$.

4. (25 points) Formally define a 1-tape Turing Machine that has three heads on the tape. (It's okay if they end up reading the same symbol.)

   SOLUTION TO PROBLEM THREE

   $(Q, \Sigma, \delta, s, h)$

   (a) Q is a set of states.
   (b) Σ is an alphabet.

(c) $s \in Q$ is the start state

(d) $h \in Q$ is the halt state - once there you are DONE.

$\delta : Q - \{h\} \times \Sigma \times \Sigma \times \Sigma \rightarrow Q \times \Sigma \cup \{R, L\}) \times (\Sigma \cup \{R, L\} \times (\Sigma \cup \{R, L\})$

The intuition is that the THREE heads are on the tapes and hence there are THREE symbols that are being seen. Having seen these two symbols the heads decide what to do. Each one moves R, moves L, or prints a symbol. They need NOT do the same thing.

5. (25 points) A *CNF-Boolean Formula* is of the form

$$(L_{11} \vee L_{12} \vee \cdots \vee L_{1k_1}) \wedge \cdots \wedge (L_{m1} \vee L_{m2} \vee \cdots \vee L_{mk_m}).$$

Describe, in terms of common data structures, a way to represent an arbitrary CNF-Boolean Formula in a computer program.

SOLUTION TO PROBLEM FOUR

6. (25 points) Let $L \in DTIME(T(n))$. Find a polynomial p such that $L^* \in DTIME(p(n)(T(n)))$. Give the algorithm that achieves this (it can use the algorithm for $L \in DTIME(T(n))$ and should be in pseudocode).

SOLUTION TO PROBLEM FIVE

The answer is $p(n) = n$ but we omit the details which is an easy adaptation for the proof we had that P is closed under *.