HW 11 CMSC 452. Morally Due April 24
HW IS TWO PAGES!

1. (5 points) What is your name? Write it clearly. Staple the HW.

2. (60 points) Let

$$IS = \{(G, k) : G \text{ has an independent set of size} \geq k\}$$

(A set of vertices, U, is an *Independent Set* if no vertex in U has an edge to any other vertex in U.)

   (a) Describe, in English — with pictures and an example — how you would, GIVEN a Boolean Formula $\phi(x_1, \ldots, x_n)$ produce a graph G and a number k such that:

$$\phi \in SAT \text{ iff } (G, k) \in IS$$

   (b) Write pseudocode for the procedure that takes a Boolean Formula ϕ and produces (G, k), as described above.

   (c) Apply your procedure to the Boolean Formula:

$$(x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee \neg x_2) \wedge x_3$$

   (d) If ϕ has L clauses and each clause has three variables, then how many vertices are in G and what is k?

   (e) Assume that 3-SAT is NP-complete. (See the next problem for definition of 3-SAT.) Find a function f such that:

$$ISF = \{G : G \text{ has } n \text{ vertices and an independent set of size} \geq f(n)\}$$

   is NP-complete.

3. (25 points)

   - Let SAT be the following problem: Given a Boolean formula, in the following form:

     $$C_1 \wedge \cdots \wedge C_L$$

     where each C_i is a disjunction ($\vee$) of literals, is the formula satisfiable?
   - Let k-SAT be the following problem: Given a Boolean formula, in the following form:

     $$C_1 \wedge \cdots \wedge C_L$$

     where each C_i is a disjunction ($\vee$) of *exactly* k literals, is the formula satisfiable?

   The above two points are definitions, NOT questions. HERE are the questions:

   (a) Show that 2-SAT is in P

   (b) Show that $SAT \leq k\text{-}SAT$, for $k \geq 3$.

   (PERMISSION: You may go to the web or elsewhere to find the answer; however, you must put it in your own words and understand your answer.)