HW 12 CMSC 452. Morally Due May 1
HW IS TWO PAGES!
SOLUTIONS

1. (5 points) What is your name? Write it clearly. Staple the HW.

2. (20 points) Let $A \in NP$. Let p_1, p_2 and B be such that:

$$A = \{x : (\exists y)[|y| = p_1(|x|) \wedge (x, y) \in B\}$$

where B is in $DTIME(p_2(n))$. B is computed by a Turing machine that has 6 symbols in the alphabet and 10 states.

Let x be an string of length n. Using the Cook-Levin Theorem, I can come up with a FORMULA ϕ such that

$x \in A$ iff $\phi \in SAT$.

How many variables does ϕ have?

SOLUTION TO PROBLEM TWO

The Cook-Levin reduction assumes that the TM runs on the string

$x\#y$ which is of length $n + p_1(n)$.

The computation runs in time $p_2(n + p_1(n))$.

Hence we have a table that is $(n + p_1(n))(p_2(n + p(1(n))$.

For each entry of the table I have the variables

$z_{ij\sigma}$ where $\sigma \in \Sigma \cup Q \times \Sigma$

There are $6 + (6 \times 10) = 66$ possible σ. Hence the number of variables is

$66(n + p_1(n))(p_2(n + p(1(n))$.

END OF SOLUTION TO PROBLEM TWO

3. (20 points) Write down the part of the Cook-Levin Formula that corresponds to the transition:

$\delta(q, a) = (p, b)$

i.e. the head does not move and prints a 'b'

SOLUTION TO PROBLEM THREE

$z_{i,j,(q,a)} \implies z_{i,j+1,(p,b)}$

KEY is that since the head does not move, nothing else changes.

END OF SOLUTION TO PROBLEM THREE

4. (30 points) Show that if $A \le B$ and $B \le C$ then $A \le C$.

SOLUTION TO PROBLEM FOUR

Assume $A \le B$ via TM M_1 that runs in $p_1(n)$ time, p_1 a poly.

Assume $B \le C$ via TM M_2 that runs in $p_2(n)$ time, p_2 a poly.

The following is a reduction $A \le C$

(a) Input(x)

(b) Run $M_1(x)$ to find y' (this takes $p_1(|x|)$ steps. Note that $|y'| \le p(|x|)$).

(c) Run $M_2(y_1)$ to find y (this take $p_2(|y'|) \le p_2(p_1(|x|))$).

(d) Output y.

$x \in A$ iff $y' \in B$ iff $y \in C$.

The procedure takes $p_2(p_1(|x|))$ steps which is a polynomial.

END OF SOLUTION TO PROBLEM FOUR

5. (30 points) Let

$COL_2 = \{G : G \text{ is 2-colorable }\}$

$COL_3 = \{G : G \text{ is 3-colorable }\}$

$COL_4 = \{G : G \text{ is 4-colorable }\}$

$COL_5 = \{G : G \text{ is 5-colorable }\}$

$PCOL_2 = \{G : G \text{ is Planar and 2-colorable }\}$

$PCOL_3 = \{G : G \text{ is Planar and 3-colorable }\}$

$PCOL_4 = \{G : G \text{ is Planar and 4-colorable }\}$

$PCOL_5 = \{G : G \text{ is Planar and 5-colorable }\}$

For each statement below you must answer TRUE or FALSE and *explain why*. You may assume the following: (1) $P \neq NP$, (2) COL_3 is NP-complete, (3) $PCOL_3$ is NP-complete, (4) SAT is NP-complete, (5) 3-SAT is NP-complete, (6) every planar graph is 4-colorable, (7) if $A \le B$ and $B \le C$ then $A \le C$.

As usual, $A \le B$ means that there is a function f computable in poly time such that, for all x

$$x \in A \text{ iff } f(x) \in B$$

(a) $COL_2 \le COL_3$

(b) $COL_3 \le COL_2$

(c) $COL_3 \le COL_4$

(d) $COL_4 \le COL_3$

(e) $PCOL_3 \le COL_4$

(f) $COL_3 \le PCOL_4$

SOLUTION TO PROBLEM FIVE

(a) $COL_2 \le COL_3$: TRUE. $COL_2 \in P$. Anything in P is $\le$ anything.

(b) $COL_3 \le COL_2$: FALSE. Since $COL_2 \in P$, if this was true then $COL_3 \in P$ but we are assuming $P \neq NP$.

(c) $COL_3 \le COL_4$: TRUE. The reduction is easy: given G, let G' be G with one more vertex added and connected to all vertices. G is 3-colorable IFF G' is 3-colorable.

(d) $COL_4 \le COL_3$: TRUE. The reduction is INSANE: By Cook-Levin theorem $COL_4 \le SAT$. By COL_3 being NP-complete, $SAT \le COL_3$. Since reductions are transitive, $COL_4 \le COL_3$.

(e) $COL_3 \le PCOL_4$. FALSE. $PCOL_4$ is in P since ALL graphs are 4-colorable. If true this would imply $P = NP$.

END OF SOLUTION TO PROBLEM FIVE