HW 13 CMSC 452. Morally Due May 8

1. (5 points) What is your name? Write it clearly. Staple the HW.

2. (30 points) Let f be a function. The *image of f* is the set of all y such that there is some x where $f(x) = y$. Formally

$$\text{image}(f) = \{y : (\exists x \in A)[f(x) = y]\}$$

   For each of the following either say TRUE or FALSE or UNKNOWN TO SCIENCE. If TRUE then prove it, if FALSE then you **do not** have to prove it, if UNKNOWN TO SCIENCE, you don't have to resolve it.

   (a) Let f be a computable function such that

   $$(\forall x, y)[x < y \rightarrow f(x) < f(y)]$$

   Then the image of f is computable.

   (b) Let f be a computable function such that

   $$(\forall x, y)[x < y \rightarrow f(x) \leq f(y)]$$

   Then the image of f is computable.

   (c) Let f be computable in polynomial time. Then the image of f is in P

3. (35 points) Show that there exists a decidable set that is **not** in $\bigcup_{a=1}^{\infty} DTIME(2^{n^a})$

4. (30 points) Let $COUNTSAT$ be the **function** that takes a boolean formula and outputs **the number** of satisfying assignments it has. (The answer could be 0.)

   (a) True or False: If $COUNTSAT$ can be computed in polytime, then $P = NP$. In either case justify your answer.

   (b) Write an algorithm for $COUNTSAT$.

   (c) How fast does your algorithm run (express as a function of n, the number of variables).