Homework 3 Morally Due Feb 19
THIS HOMEWORK IS TWO PAGES LONG!!!!!!!!!!!!!!!

1. (30 points) The alphabet is $\{a, b\}$. Let $n \geq 0$ and let

$$L_n = \{a, b\}^* a \{a, b\}^n$$

(so the $(n+1)$th letter from the end is a).

   (a) (15 points) Draw a DFA for L_n when $n = 2$. Describe the DFA for L_n for any general n. How many states does L_n have in general as a function of n?

   (b) (15 points) Draw an NFA for L_n for any general n. You may use DOT DOT DOT and other shortcuts. How many states does it have as a function of n?

   (c) (0 points) THINK ABOUT proving that any DFA for L_n has LOTS of states.

2. (30 points) Use the conventions about representing numbers and sets established in class. Your DFA's should have ACCEPT states (labelled A), REJECT states (labelled R), and STUPID states (labelled S).

   (a) (15 points) Draw a DFA for

$$\{(x, A) \mid x + 1 \in A\}$$

   How many states does it have?

   (b) (15 points) For all n draw a DFA (you may use DOT DOT DOT) for

$$L_n = \{(x, A) \mid x + n \in A\}$$

   How many states does it have as a function of n?

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3. (20 points) Note that $47 \times 101 = 4747$. Let

$$L = \{a^n \mid n \not\equiv 0 \pmod{4747}\}.$$

There is clearly a DFA for L with 4747 states.

(a) (10 points) Prove that ANY DFA for L has to have ≥ 4747 states.

(b) (10 ponts) Prove or Disprove: There is an NFA for L with < 4747 states.

SOLUTION TO PROBLEM THREE

a) Assume that there is a DFA for L with < 4747 states. Input a^{4747} to this DFA. In its run there must be a repeated state. Hence there exist numbers i and j with $1 \leq i < j \leq 4747$ such that

a^i and a^j both end up in state q.

Then $a^i a^{4747-j} = a^{4747+i-j} \neq a^{4747}$ and $a^j a^{4747-j} = a^{4747}$ both end up in the same state q. But the first string should be accepted and the second one rejected. This is a contradiction.

b) Omitted for now- will post later. I have my reasons.

b) YES there is an NFA for L with MUCH LESS than 4747 states.

Let $n \not\equiv 4747$. Note that n CANNOT be BOTH $\equiv 0 \pmod{101}$ and $\equiv 0 \pmod{47}$ (if it was then it would be $\equiv 0 \pmod{4747}$). Hence either

- There exists $i \in \{1, \ldots, 46\}$, $n \equiv i \pmod{47}$, or
- There exists $i \in \{1, \ldots, 100\}$, $n \equiv i \pmod{101}$.

Hence your NFA does the following: two e-transistions from the start state: (1) one of them goes to a DFA that accepts iff $n \not\equiv 0 \pmod{47}$ (this takes 47 states), (2) the other goes to a DFA that accepts iff $n \not\equiv 0 \pmod{101}$ (this takes 101 states)

Hence there is an NFA for L with 148 states plus the start state, so 149 states MUCH less than 4747.

END OF SOLUTION TO PROBLEM THREE

4. (20 points) Write psuedo code for an algorithm that will, GIVEN a DFA M determine if $L(M) \neq \emptyset$ or not. (Here, $L(M)$ denotes the language that M accepts.) It must be completely self-contained, so you can't say something like *Use Kruskal's MST algorithm here and pay Clyde's Uncle Joe the Royalties* in the algorithm (HINT: that is not part of any correct answer that I know of anyway.)

SOLUTION TO PROBLEM 4

(a) Input DFA $M = (Q, \Sigma, \delta, s, F)$.

(b) $X_0 := \{s\}$

(c) $n := |Q|$.

(d) For $i := 1$ to n $X_i := X_{i-1} \cup \{\delta(q, \sigma) \mid q \in X_{i-1}, \sigma \in \Sigma\}$

(e) If $X_n \cap F \neq \emptyset$ then output $L(M) \neq \emptyset$.

(f) If $X_n \cap F = \emptyset$ then output $L(M) = \emptyset$.